

## Converting between the Forms Circles

### Objective:

- Convert the equation of a circle from Standard Form to General Form
- Convert the equation of a circle from General Form to Standard Form by completing the square

### Standard Form equation of a Circle:

$$(x-h)^2 + (y-k)^2 = r^2$$

### General Form equation of a Circle:

$$Ax^2 + By^2 + Cx + Dy + E = 0$$

### PART I: Converting from Standard Form to General Form

Step 1-Expand the binomial terms

Step 2-Combine like terms

Step 3-Move all terms to one side & write them in order.

Ex. 1: Write the following equation in General Form:

$$(x-3)^2 + (y-4)^2 = 36$$

$$\begin{aligned} (x-3)(x-3) + (y-4)(y-4) &= 36 \\ x^2 - 3x - 3x + 9 + y^2 - 4y - 4y + 16 &= 36 \\ x^2 - 6x + y^2 - 8y + 25 &= 36 \\ &\quad \quad \quad -36 \quad -36 \end{aligned}$$

$$x^2 + y^2 - 6x - 8y - 11 = 0$$

Ex. 2: Write the following equation in General Form:

$$(x+4)^2 + (y-1)^2 = 4$$

$$\begin{aligned} (x+4)(x+4) + (y-1)(y-1) &= 4 \\ x^2 + 4x + 4x + 16 + y^2 - y - y + 1 &= 4 \\ x^2 + 8x + y^2 - 2y + 17 &= 4 \\ &\quad \quad \quad -4 \quad -4 \end{aligned}$$

$$x^2 + y^2 + 8x - 2y + 13 = 0$$

Ex. 3: Write the General Form equation for the circle with center (1, -6) and radius 4.

$$\begin{aligned} (x-1)^2 + (y+6)^2 &= 16 \\ (x-1)(x-1) + (y+6)(y+6) &= 16 \\ x^2 - x - x + 1 + y^2 + 6y + 6y + 36 &= 16 \\ x^2 - 2x + y^2 + 12y + 37 &= 16 \\ &\quad \quad \quad -16 \quad -16 \end{aligned}$$

$$x^2 + y^2 - 2x + 12y + 21 = 0$$

## Converting between the Forms Circles

### PART II: Converting from General Form to Standard Form by Completing the Square

Step 1-Group the x and y terms and move the constant to the other side of the equal sign.

Step 2-Complete the square for the x group and for the y group. (Fill in the blank with  $(\frac{b}{2})^2$ .)

Step 3-Balance the equation by adding  $(\frac{b}{2})^2$  to both sides of the equation.

Step 4-Factor each group and write each group as a binomial squared.  $(x+\frac{b}{2})^2$

**Ex. 4:** Write the equation in standard form by completing the square. Then identify the center and the radius.

$$x^2 + y^2 + 2x - 4y - 11 = 0$$

$$(x^2 + 2x + \underline{1}) + (y^2 - 4y + \underline{4}) = 11 + \underline{1} + \underline{4}$$

$$(\frac{2}{2})^2 = (1)^2 = 1 \quad (\frac{-4}{2})^2 = (-2)^2 = 4$$

$$(x+1)^2 + (y-2)^2 = 16$$

**Ex. 5:** Write the equation in standard form by completing the square. Then identify the center and the radius.

$$x^2 + y^2 - 4x + 2y - 4 = 0$$

$$(x^2 - 4x + \underline{4}) + (y^2 + 2y + \underline{1}) = 4 + \underline{4} + \underline{1}$$

$$(\frac{-4}{2})^2 = (-2)^2 = 4 \quad (\frac{2}{2})^2 = (1)^2 = 1$$

$$(x-2)^2 + (y+1)^2 = 9$$

**Ex. 6:** Write the equation in standard form by completing the square. Then identify the center and the radius.

$$x^2 + y^2 - 6x - 10y = 2$$

$$(x^2 - 6x + \underline{9}) + (y^2 - 10y + \underline{25})$$

$$= 2 + \underline{9} + \underline{25}$$

$$(\frac{-6}{2})^2 = (-3)^2 = 9 \quad (\frac{-10}{2})^2 = (-5)^2 = 25$$

$$(x-3)^2 + (y-5)^2 = 36$$

### In Class Practice:

Write each equation in general form.

1)  $(x+7)^2 + (y+3)^2 = 9$

2)  $(x-5)^2 + (y+4)^2 = 25$

Write each equation in standard form.

3)  $x^2 + y^2 + 2x + 4y - 20 = 0$

4)  $x^2 + y^2 - 4y = 10$

## In class practice

$$1) (x+7)^2 + (y+3)^2 = 9$$

$$(x+7)(x+7) + (y+3)(y+3) = 9$$

$$x^2 + 7x + 7x + 49 + y^2 + 3y + 3y + 9 = 9$$

$$x^2 + 14x + 58 + y^2 + 6y = 9$$

$$\boxed{x^2 + y^2 + 14x + 6y + 49 = 0}$$

$$2) (x-5)^2 + (y+4)^2 = 25$$

$$(x-5)(x-5) + (y+4)(y+4) = 25$$

$$x^2 - 5x - 5x + 25 + y^2 + 4y + 4y + 16 = 25$$

$$x^2 - 10x + 41 + y^2 + 8y = 25$$

$$\boxed{x^2 + y^2 - 10x + 8y + 16 = 0}$$

$$3) x^2 + y^2 + 2x + 4y - 20 = 0$$

$$(x^2 + 2x + \underline{1}) + (y^2 + 4y + \underline{4}) = 20 + \underline{1+4}$$

$$\left(\frac{2}{2}\right)^2 = (1)^2 = 1 \quad \left(\frac{4}{2}\right)^2 = (2)^2 = 4$$

$$\boxed{(x+1)^2 + (y+2)^2 = 25}$$

$$4) x^2 + y^2 - 4y = 10$$

$$(x^2 + \underline{0}) + (y^2 - 4y + \underline{4}) = 10 + \underline{0+4}$$

$$\left(\frac{0}{2}\right)^2 = 0 \quad \left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$$

$$\boxed{x^2 + (y-2)^2 = 14}$$