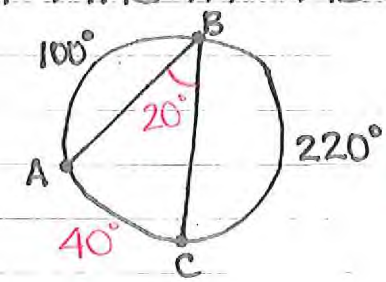
More on Inscribed Angles

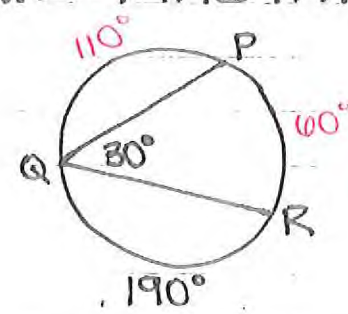
We have already learned that the measure of an inscribed angle is half the measure of its arc.

Ex.1: Find $m\angle ABC$.



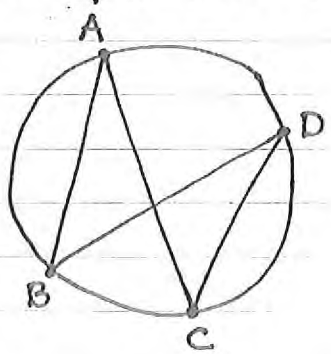
20°

Ex.2: Find $m\widehat{PQ}$.



110°

What happens when two inscribed angles intercept the same arc? Let's find out!



i) $\angle ABD$ is intercepted by what arc? $\widehat{AD}$

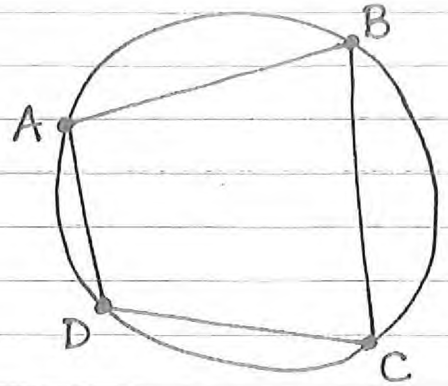
ii) $\angle ACD$ is intercepted by what arc? $\widehat{AD}$

iii) Let $m\angle ABD = 20°$. What is $m\widehat{AD}$? 40°
What is $m\angle ACD$? 20°

iv) consider $m\angle BAC = 10°$. What is $m\angle BDC$? 10° Explain how you know.
They both are intercepted by $\widehat{BC}$, so the angles are both half that arc.

Conjecture: If two inscribed angles of a circle intercept the same arc, then the angles are congruent.

When can a quadrilateral be inscribed in a circle? Let's find out!



Quadrilateral ABCD has been inscribed in the circle.

$* \; m\overset{\frown}{BCD} = 2 \cdot m\angle A$

$* \; m\overset{\frown}{DAB} = 2 \cdot m\angle C$

$m\overset{\frown}{BCD} + m\overset{\frown}{DAB} = 360°$

Let's use the substitution property:

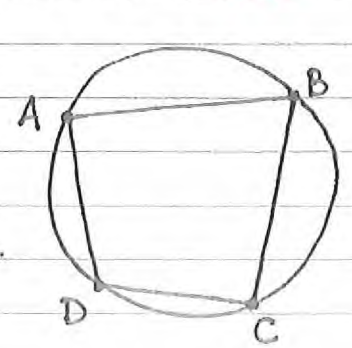
$$m\overset{\frown}{BCD} + m\overset{\frown}{DAB} = 360°$$

$$* \; 2 \cdot m\angle A + 2 \cdot m\angle C = 360°$$

$$2(m\angle A + m\angle C) = 360°$$

$$m\angle A + m\angle C = 180°$$

Conjecture: A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary, which means they add to 180°.



$m\angle A + m\angle C = 180°$

$m\angle B + m\angle D = 180°$