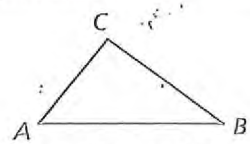
TRIANGLE + PERPENDICULAR BISECTOR THEOREM NOTES

The Triangle Sum Theorem

The sum of the angle measures in a triangle is 180°.

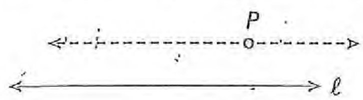


$m\angle A + m\angle B + m\angle C = 180°$

The proof of the Triangle Sum Theorem depends upon a postulate known as the Parallel Postulate.

The Parallel Postulate

Through a point P not on a line ℓ, there is exactly one line parallel to ℓ.

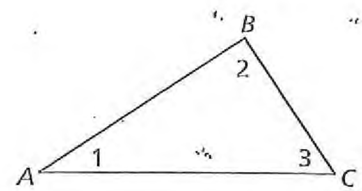


Prove the Triangle Sum Theorem.

The sum of the angle measures in a triangle is 180°.

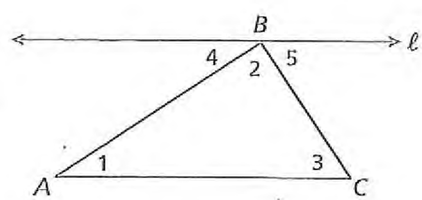
Given: $\triangle ABC$

Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180°$



A Understand the plan for the proof.

Draw a line through B that is parallel to $\overline{AC}$. This creates three angles that form a straight angle, so the sum of their measures is 180°. Use the fact that alternate interior angles have the same measure to conclude that the sum of the measures of the angles in a triangle is 180°.

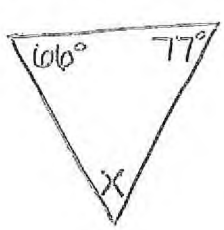


B Complete the proof.

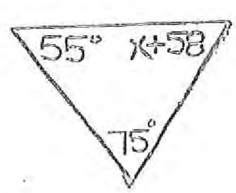
Statements	Reasons
0. $\triangle ABC$	0. Given
1. Draw ℓ through point B parallel to $\overline{AC}$.	1. The Parallel Postulate
2. $m\angle 4 = m\angle 1$ and $m\angle 5 = m\angle 3$	2.
3. $m\angle 4 + m\angle 2 + m\angle 5 = 180°$	3. Angle Addition Postulate and definition of straight angle
4.	4.

EXAMPLES: Find X.

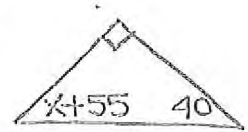
1)



2)

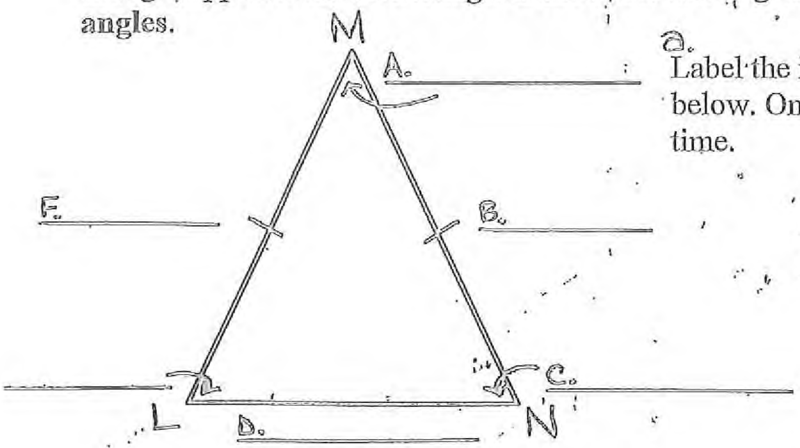


3)



sosceles Triangle Theorem

An **isosceles triangle** is a triangle with at least two congruent sides. The congruent sides are called the **legs** of the triangle. The angle formed by the legs is the **vertex angle**. The side of the triangle opposite the vertex angle is the **base**. The angles that have the base as a side are the **base angles**.

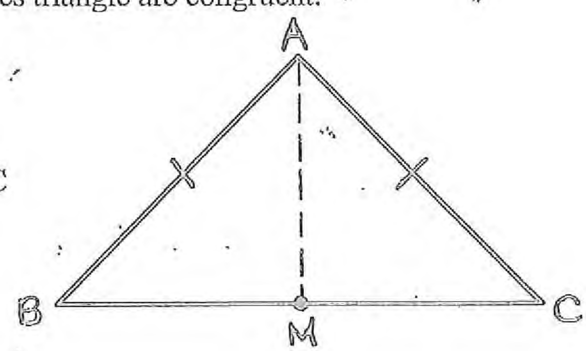


a. Label the isosceles triangle using the words below. One word may be used more than one time.

Vertex angle
Base
Leg
Base angle

Theorem: If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

You will now prove that the base angles of an isosceles triangle are congruent.

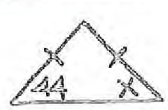


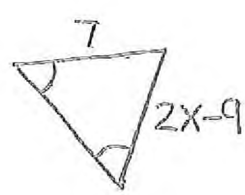
Proof:

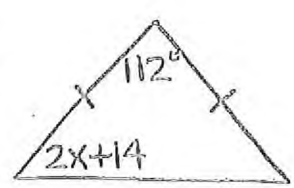
Given: $AB \cong AC$ and M is the midpoint of BC
Prove: $\angle B \cong \angle C$

Statement	Reason
	Definition of Midpoint
	Reflexive Property of Congruence
$\triangle AMC \cong \triangle AMB$	
	CPCTC

EXAMPLES: Find X.

1)


2)


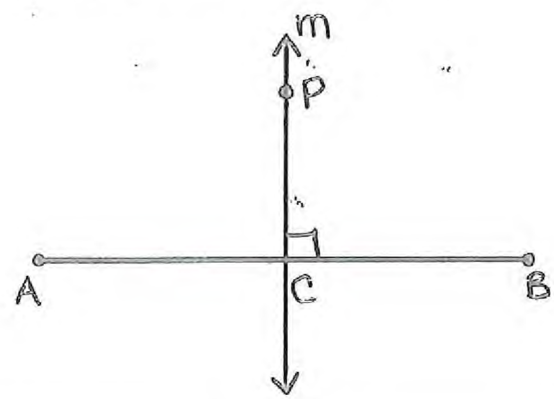
3)


The **perpendicular bisector** of a line segment is a line that is perpendicular to the segment at the segment's midpoint.

a. Construct the perpendicular bisector of $\overline{AB}$.
b. Create a point P that is on the perpendicular bisector.
c. Use wax paper and trace the distance between A and P.
d. Compare that distance to the distance between B and P.
e. What do you notice?

Theorem: If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

You will now prove the above theorem.



Proof:

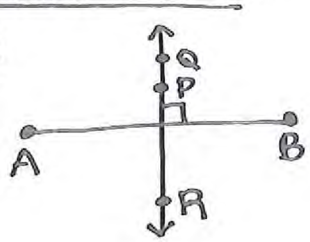
Given: P is on the perpendicular bisector m of $\overline{AB}$.

Prove: $\overline{PA} \cong \overline{PB}$

Statement	Reason
P is on the perpendicular bisector m of $\overline{AB}$	
C is the midpoint	Given
	Definition of Midpoint
$\angle PCB \cong \angle PCA$	
	Reflexive Property of Congruence
	SAS Congruence
	CPCTC

EXAMPLES:

1)

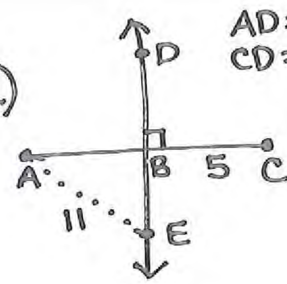


$\overline{AP} \cong$ ____

$\overline{RB} \cong$ ____

$\overline{QA} \cong$ ____

2)



AD=12X+2

CD=26+10X

$\overline{AB}$ = ____

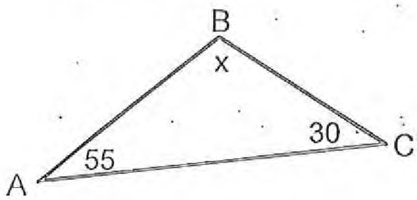
$\overline{CE}$ = ____

X = ____

1) *Example: Prove the following.*

Given: $\triangle ABC$

Prove: $x = 95$

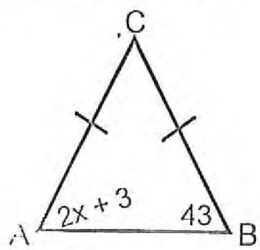


Statements	Reasons

2) *Example: Prove the following.*

Given: $\overline{AC} \cong \overline{BC}$

Prove: $x = 20$

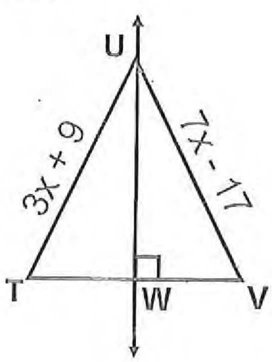


Statements	Reasons

3) *Example: Prove the following.*

Given: $\overleftrightarrow{WU}$ is the perpendicular bisector of $\overline{TV}$

Prove: $x = 6.5$



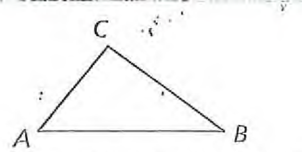
Statements	Reasons

TRIANGLE + PERPENDICULAR BISECTOR THEOREM NOTES

#19

The Triangle Sum Theorem

The sum of the angle measures in a triangle is 180°.

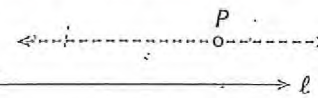


$m\angle A + m\angle B + m\angle C = 180°$

The proof of the Triangle Sum Theorem depends upon a postulate known as the Parallel Postulate.

The Parallel Postulate

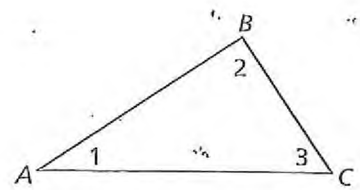
Through a point P not on a line ℓ, there is exactly one line parallel to ℓ.



Prove the Triangle Sum Theorem.

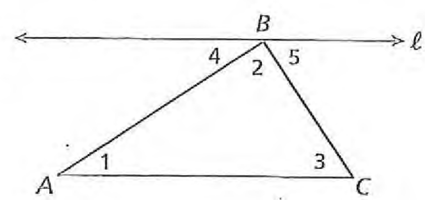
The sum of the angle measures in a triangle is 180°.

Given: $\triangle ABC$
Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180°$



A Understand the plan for the proof.

Draw a line through B that is parallel to $\overline{AC}$. This creates three angles that form a straight angle, so the sum of their measures is 180°. Use the fact that alternate interior angles have the same measure to conclude that the sum of the measures of the angles in a triangle is 180°.

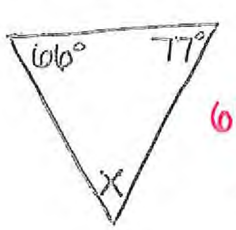


B Complete the proof.

Statements	Reasons
~~0. $\triangle ABC$~~	~~0. Given~~
1. Draw ℓ through point B parallel to $\overline{AC}$.	1. The Parallel Postulate
2. $m\angle 4 = m\angle 1$ and $m\angle 5 = m\angle 3$	2. Alternate Interior Angles Theorem
3. $m\angle 4 + m\angle 2 + m\angle 5 = 180°$	3. Angle Addition Postulate and definition of straight angle
4. $m\angle 1 + m\angle 2 + m\angle 3 = 180$	4. Substitution Prop of =

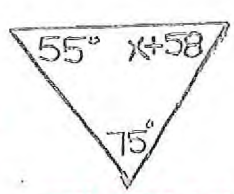
EXAMPLES: Find X.

1)



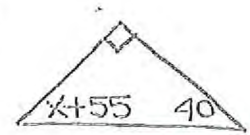
$66 + 77 + X = 180$
$143 + X = 180$
$\boxed{X = 37}$

2)



$55 + 75 + X + 58 = 180$
$X + 188 = 180$
$\boxed{X = -8}$

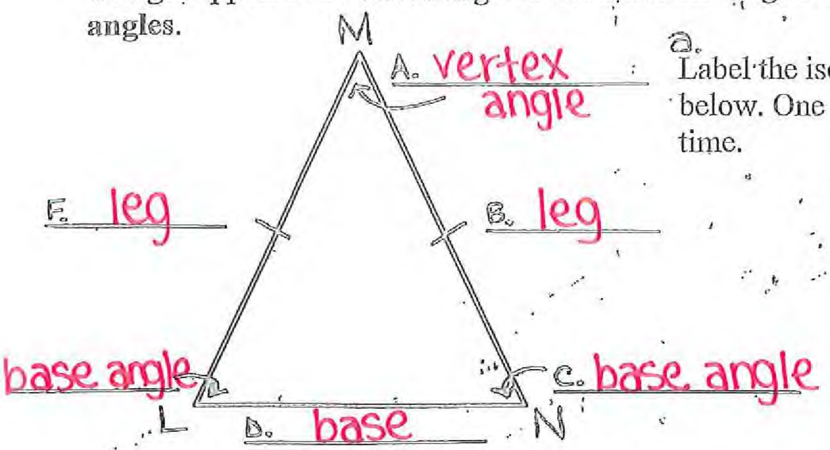
3)



$X + 55 + 40 + 90 = 180$
$X + 185 = 180$
$\boxed{X = -5}$

Isosceles Triangle Theorem

An **isosceles triangle** is a triangle with at least two congruent sides. The congruent sides are called the **legs** of the triangle. The angle formed by the legs is the **vertex angle**. The side of the triangle opposite the vertex angle is the **base**. The angles that have the base as a side are the **base angles**.

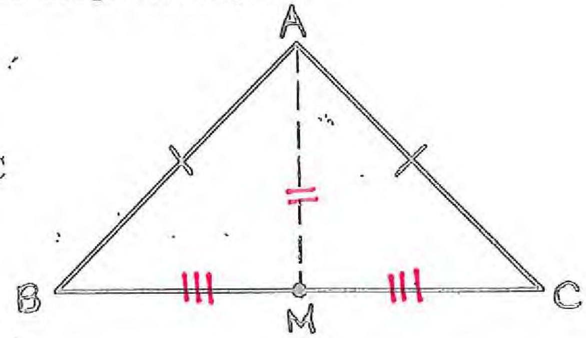


a. Label the isosceles triangle using the words below. One word may be used more than one time.

Vertex angle
Base
Leg
Base angle

Theorem: If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

You will now prove that the base angles of an isosceles triangle are congruent.



Proof:

Given: $AB \cong AC$ and M is the midpoint of BC
Prove: $\angle B \cong \angle C$

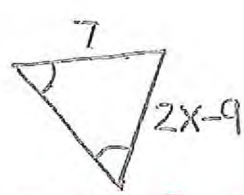
Statement	Reason
$AB \cong AC$ M is the midpoint of BC	Given
$BM \cong MC$	Definition of Midpoint
$AM \cong AM$	Reflexive Property of Congruence
$\triangle AMC \cong \triangle AMB$	SSS Congruence Theorem
$\angle B \cong \angle C$	CPCTC

EXAMPLES: Find x.

1)

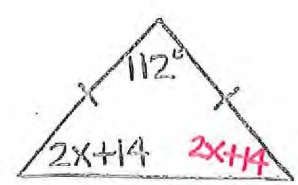
$\boxed{x = 44}$

2)



$7 = 2x - 9$
$16 = 2x \rightarrow \boxed{x = 8}$

3)



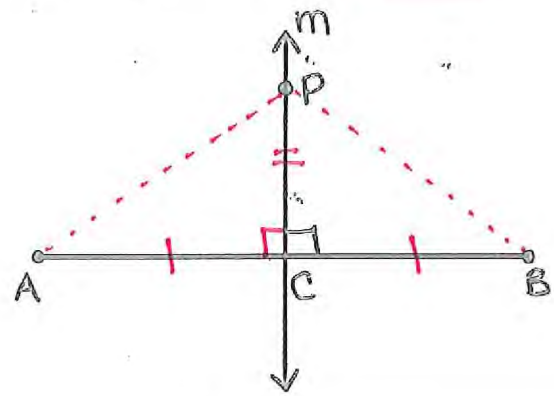
$112 + 2x + 14 + 2x + 14 = 180$
$140 + 4x = 180$
$4x = 40 \rightarrow \boxed{x = 10}$

The **perpendicular bisector** of a line segment is a line that is perpendicular to the segment at the segment's midpoint.

a. Construct the perpendicular bisector of $\overline{AB}$.
b. Create a point P that is on the perpendicular bisector.
c. Use wax paper and trace the distance between A and P.
d. Compare that distance to the distance between B and P.
e. What do you notice?

Theorem: If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

You will now prove the above theorem.



Proof:

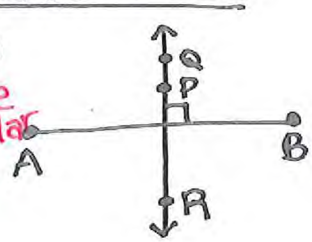
Given: P is on the perpendicular bisector *m* of $\overline{AB}$.
Prove: $\overline{PA} \cong \overline{PB}$

Statement	Reason
P is on the perpendicular bisector *m* of $\overline{AB}$	Given
C is the midpoint	Given
$\overline{AC} \cong \overline{BC}$	Definition of Midpoint
$\angle PCB \cong \angle PCA$	$\overleftrightarrow{PC} \perp \overline{AB}$
$\overline{PC} \cong \overline{PC}$	Reflexive Property of Congruence
$\Delta PCA \cong \Delta PCB$	SAS Congruence
$\overline{PA} \cong \overline{PB}$	CPCTC

EXAMPLES:

1)

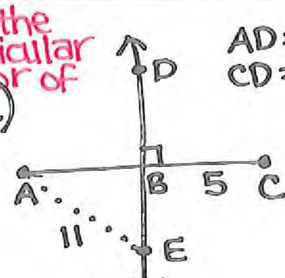
$\overleftrightarrow{QR}$ is the perpendicular bisector of $\overline{AB}$



$\overline{AP} \cong$ $\overline{BP}$
$\overline{RB} \cong$ $\overline{RA}$
$\overline{QA} \cong$ $\overline{QB}$

2)

$\overleftrightarrow{DE}$ is the perpendicular bisector of $\overline{AC}$



AD = 12X + 2
CD = 26 + 10X

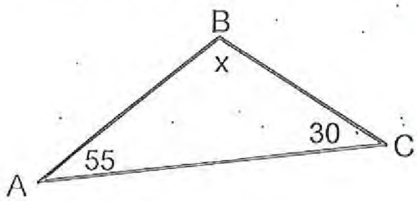
12X + 2 = 26 + 10X
2X + 2 = 26
2X = 24
X = 12

$\overline{AB}$ = 5
$\overline{CE}$ = 11
X = 12

1) *Example: Prove the following.*

Given: $\triangle ABC$

Prove: $x = 95$

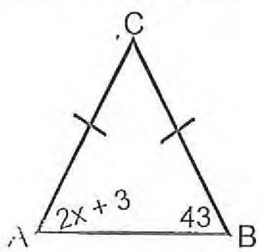


Statements	Reasons
$\triangle ABC$	Given
$m\angle A + m\angle B + m\angle C = 180$	Triangle Sum Theorem
$55 + x + 30 = 180$	Substitution Prop of =
$85 + x = 180$	simplify
$x = 95$	subtraction Prop of =

2) *Example: Prove the following.*

Given: $\overline{AC} \cong \overline{BC}$

Prove: $x = 20$

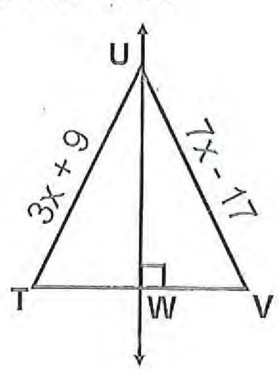


Statements	Reasons
$\overline{AC} \cong \overline{BC}$	Given
$m\angle A = m\angle B$	Isosceles Triangle Theorem
$2x+3 = 43$	substitution Prop of =
$2x = 40$	Subtraction Prop of =
$x = 20$	Division Prop of =

3) *Example: Prove the following.*

Given: $\overleftrightarrow{WU}$ is the perpendicular bisector of $\overline{TV}$

Prove: $x = 6.5$



Statements	Reasons
$\overleftrightarrow{WU}$ is the perpendicular bisector of $\overline{TV}$	Given
$UT \cong UV$	Perpendicular Bisector Theorem
$3x+9 = 7x-17$	substitution Prop of =
$-4x+9 = -17$	subtraction Prop of =
$-4x = -26$	subtraction Prop of =
$x = 6.5$	Division Prop of =