

# Conditional Probability

#21

Conditional probabilities are found when one event has already occurred and a second event is being analyzed. Conditional probability is denoted \_\_\_\_\_ and is read as "the probability of A given B".

To find conditional probabilities:

~~Ex. 1: Two marbles are drawn without replacement with 3 white, 2 green, 2 red, and one blue. Find the probability that:~~

- ~~a) The second marble is white given that the first marble is blue \_\_\_\_\_~~
- ~~b) The second marble is blue given that the first marble is blue \_\_\_\_\_~~

Ex. 2: If a single fair dice is rolled, find the probability of a rolling a 5 given that the number rolled is odd. \_\_\_\_\_

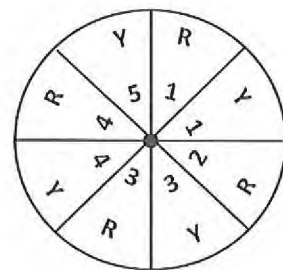
Ex. 3: A dice is tossed. Find  $P(\text{less than } 5 \mid \text{even})$ . \_\_\_\_\_

Ex. 4: A box contains three blue cards, 5 red cards, and four white cards. If one card is drawn at random, find:

- a)  $P(\text{blue} \mid \text{not white})$  \_\_\_\_\_
- b) The probability of drawing a red card given that the card is not white \_\_\_\_\_

Ex. 5: A spinner with dial marked as shown is spun once. Find:

- a) The probability that it points to an even number given the area is red \_\_\_\_\_
- b)  $P(\text{odd number} \mid \text{yellow})$  \_\_\_\_\_



Ex. 6: A number is selected at random from the set  $\{1, 2, 3, 4, 5, 6, 7, 8\}$ . Find:

- a)  $P(\text{odd})$  \_\_\_\_\_
- b) The probability that the number is a multiple of 2 given the number is less than 7 \_\_\_\_\_

Ex. 7: In New York State, 48% of all teenagers own a skateboard and 39% of all teenagers own a skateboard and roller blades. What is the probability that a teenager owns roller blades given that the teenager owns a skateboard? \_\_\_\_\_

Ex. 8: In the United States, 56% of all children get an allowance and 41% of all children get an allowance and do household chores. What is the probability that a child does household chores given that the child gets an allowance? \_\_\_\_\_

## PRACTICE

- 1) In Europe, 88% of all households have a television. 51% of all households have a television and a VCR. What is the probability that a household has a VCR given that it has a television?
- 2) At a middle school, 18% of all students play football and basketball and 32% of all students play football. What is the probability that a student plays basketball given that the student plays football?
- 3) You select one card from a standard deck of 52 cards.
  - a. What is the probability that the card that you select is a face card given that the card is a diamond?
  - b. What is the probability that the card that you select is an Ace given that the card is a black card?
- 4) You roll a number cube.
  - a. What is the probability that you roll a number less than 3 given that the number is odd?
  - b. What is the probability that you roll an even number given that the number is greater than 2?
- 5) A spinner has 7 equal sectors lettered A, B, C, D, E, F, G. The sectors with A, B, C, and D on them are yellow. The sectors with E, F, and G on them are green.
  - a. What is the probability that the spinner lands on a vowel given that the sector is green?
  - b. What is the probability that the spinner lands on a yellow sector given that the letter is a consonant?
- 6) What is the probability of choosing a queen, king, or an ace from a standard deck of 52 playing cards?
- 7) What is the probability of choosing a jack from a standard deck of 52 playing cards, leaving that card out of the deck and then choosing a face card?
- 8) What is the probability of choosing a red card from a standard deck of 52 playing cards, placing it back in the deck and then choosing a spade?
- 9) What is the probability of choosing a card with an odd number on it?
- 10) Consider the sample space:  $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ 

Let...	$A = \{\text{multiples of } 3\}$	$C = \{\text{numbers greater than } 7\}$
	$B = \{1, 6, 7, 9\}$	$D = \{2, 4, 5, 6, 7, 11, 12\}$

  - a. Find  $A \cup B$ . \_\_\_\_\_
  - b. Find  $\sim C$ . \_\_\_\_\_
  - c. Find  $B \cap D$ . \_\_\_\_\_
  - d. Find  $\sim(B \cup C)$ . \_\_\_\_\_
  - e. Find  $\sim A \cap D$ . \_\_\_\_\_

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Conditional probabilities are found when one event has already occurred and a second event is being analyzed. Conditional probability is denoted  $P(A|B)$  and is read as "the probability of A given B".

To find conditional probabilities:

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

~~Ex. 1: Two marbles are drawn without replacement with 3 white, 2 green, 2 red, and one blue. Find the probability that:~~

- ~~a) The second marble is white given that the first marble is blue \_\_\_\_\_  
b) The second marble is blue given that the first marble is blue \_\_\_\_\_~~

Ex. 2: If a single fair dice is rolled, find the probability of a rolling a 5 given that the number rolled is odd.  $\frac{1}{3}$  (1 2 3 4 5 6)

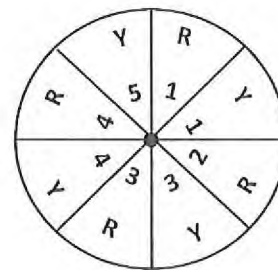
Ex. 3: A dice is tossed. Find  $P(\text{less than 5} | \text{even})$ .  $\frac{2}{3}$  (1 2 3 4 5 6)

Ex. 4: A box contains three blue cards, 5 red cards, and four white cards. If one card is drawn at random, find:

- a)  $P(\text{blue} | \text{not white})$   $\frac{3}{8}$   
b) The probability of drawing a red card given that the card is not white  $\frac{5}{8}$

Ex. 5: A spinner with dial marked as shown is spun once. Find:

- a) The probability that it points to an even number given the area is red  $\frac{2}{4}$   
b)  $P(\text{odd number} | \text{yellow})$   $\frac{3}{4}$



Ex. 6: A number is selected at random from the set {1,2,3,4,5,6,7,8}. Find:

- a)  $P(\text{odd})$   $\frac{4}{8}$   
b) The probability that the number is a multiple of 2 given the number is less than 7  $\frac{3}{6}$

Ex. 7: In New York State, 48% of all teenagers own a skateboard and 39% of all teenagers own a skateboard and roller blades. What is the probability that a teenager owns roller blades given that the teenager owns a skateboard?  $\frac{39}{48} = 81\%$

Ex. 8: In the United States, 56% of all children get an allowance and 41% of all children get an allowance and do household chores. What is the probability that a child does household chores given that the child gets an allowance?  $\frac{41}{56} = 73\%$



## PRACTICE

1) In Europe, 88% of all households have a television. 51% of all households have a television and a VCR. What is the probability that a household has a VCR given that it has a television?  $\frac{51\%}{88\%} = 58\%$

2) At a middle school, 18% of all students play football and basketball and 32% of all students play football. What is the probability that a student plays basketball given that the student plays football?  $\frac{18\%}{32\%} = 56\%$

3) You select one card from a standard deck of 52 cards.

a. What is the probability that the card that you select is a face card given that the card is a diamond?  $\frac{3}{13}$

b. What is the probability that the card that you select is an Ace given that the card is a black card?  $\frac{2}{26}$

4) You roll a number cube.

a. What is the probability that you roll a number less than 3 given that the number is odd?  $\frac{1}{3}$  (1, 2, 3, 4, 5, 6)

b. What is the probability that you roll an even number given that the number is greater than 2?  $\frac{3}{4}$  (1, 2, 3, 4, 5, 6)

5) A spinner has 7 equal sectors lettered A, B, C, D, E, F, G. The sectors with A, B, C, and D on them are yellow. The sectors with E, F, and G on them are green.

a. What is the probability that the spinner lands on a vowel given that the sector is green?  $\frac{1}{3}$

b. What is the probability that the spinner lands on a yellow sector given that the letter is a consonant?  $\frac{3}{5}$

6) What is the probability of choosing a queen, king, or an ace from a standard deck of 52 playing cards?  $\frac{12}{52}$

7) What is the probability of choosing a jack from a standard deck of 52 playing cards, leaving that card out of the deck and then choosing a face card?  $\frac{4}{52} \cdot \frac{11}{51} = \frac{44}{2652}$

8) What is the probability of choosing a red card from a standard deck of 52 playing cards, placing it back in the deck and then choosing a spade?  $\frac{26}{52} \cdot \frac{13}{52} = \frac{338}{2704}$

9) What is the probability of choosing a card with an odd number on it?  $\frac{16}{52}$

10) Consider the sample space: {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

Let... A = {multiples of 3} = {3, 6, 9, 12}  
B = {1, 6, 7, 9}

C = {numbers greater than 7} = {8, 9, 10, 11, 12}  
D = {2, 4, 5, 6, 7, 11, 12}

a. Find A ∪ B. {1, 3, 6, 7, 9, 12}

b. Find ~C. {1, 2, 3, 4, 5, 6, 7}

c. Find B ∩ D. {6, 7}

d. Find ~(B ∪ C). {1, 6, 7, 8, 9, 10, 11, 12} = {2, 3, 4, 5}

e. Find ~A ∩ D. {1, 2, 4, 5, 7, 8, 10, 11} ∩ {2, 4, 5, 6, 7, 11, 12} = {2, 4, 5, 7, 11}

