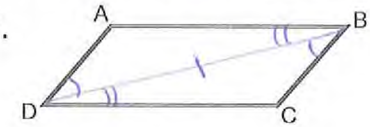
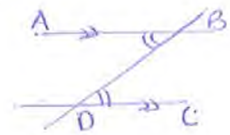
Parallelogram Theorems ∗A Parallelogram is a quadrilateral with 2 pair of parallel sides. Key

In a parallelogram, opposite sides are congruent.

Proof: Given: ABCD is a parallelogram.
Prove: $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{BC}$

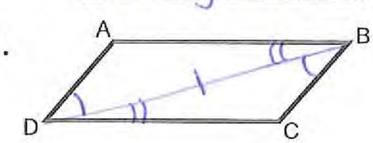




Statement	Reason
ABCD is a parallelogram	Given
Draw $\overline{BD}$	Through any two points there exists exactly on line
$\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$	Definition of Parallelogram
<ADB ≅ <CBD and <ABD ≅ <CDB	Alternate Interior Angles Theorem
$\overline{DB} \cong \overline{DB}$	Reflexive Property
$\triangle DAB \cong \triangle BCD$	ASA Congruence Theorem
$\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{BC}$	CPCTC

In a parallelogram, opposite angles are congruent.

Proof: Given: ABCD is a parallelogram.
Prove: <A ≅ <C



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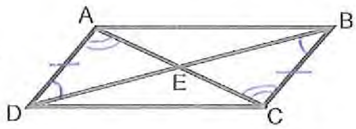
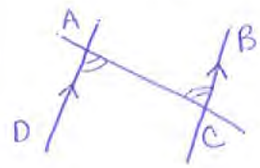
<ADB ≅ <CBD and <ABD ≅ <CDB	Alternate Interior Angles Theorem
$\overline{DB} \cong \overline{DB}$	Reflexive Property
$\triangle ABD \cong \triangle CDB$	ASA Congruence Theorem
$\angle A \cong \angle C$	CPCTC

Similarly we would prove that <B ≅ <D.

In a parallelogram, the diagonals bisect each other.

Proof: Given: ABCD is a parallelogram.
Prove: $\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$

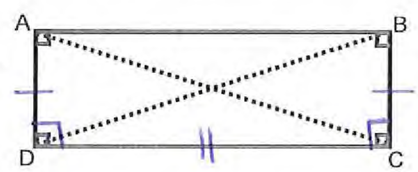


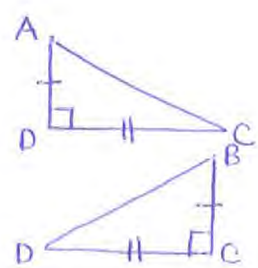


Statement	Reason
ABCD is a parallelogram.	Given
$\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$	Definition of Parallelogram
$\overline{AD} \cong \overline{BC}$	Property of Parallelograms
<ADB ≅ <CBD and <ACB ≅ <CAD	Alternate Interior Angles Theorem
$\triangle$ AED ≅ $\triangle$CEB	ASA Congruence Theorem
$\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$	CPCTC

A rectangle is a parallelogram with congruent diagonals.

Proof: Given: ABCD is a rectangle.
Prove: ABCD is a parallelogram; $\overline{AC} \cong \overline{BD}$





First prove that ABCD is a parallelogram. Since ABCD is a rectangle, <A and <C are right angles. So, <A ≅ <C because all right angles are congruent. By similar reasoning, <B ≅ <D. Therefore, ABCD is a parallelogram by the Opposite Angles Criterion for a Parallelogram.

Now prove that the diagonals are congruent. Since ABCD is a parallelogram, $\overline{AD} \cong \overline{BC}$ because of a property of parallelograms. Also, $\overline{DC} \cong \overline{DC}$ by the Reflexive property. By the definition of a rectangle, <D and <C are right angles, so <D ≅ <C because all right angles are congruent.

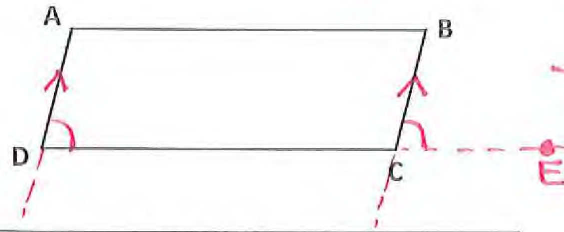
Therefore, $\triangle$ADC ≅ $\triangle$BCD by SAS Congruence Thm and $\overline{AC} \cong \overline{BD}$ by CPCTC.

Parallelogram Theorems Continued

Consecutive angles in a parallelogram are Supplementary

Proof

Given: ABCD is a parallelogram.
Prove: <ADC + <BCD = 180



Statements	Reasons
ABCD is a parallelogram	Given
Draw line DE as a continuation of DC	Construction: Extending a line segment
~~AB II DC~~ AD ∥ $\overline{BC}$	Definition of a Parallelogram
<ADC≅<BCE	Corresponding Angles Theorem
<BCE and <BCD form a linear pair	Definition of a Linear Pair
$\angle BCE + \angle BCD = 180$	Linear Pair Theorem
$\angle ADC + \angle BCD = 180$	Substitution Prop of Equality

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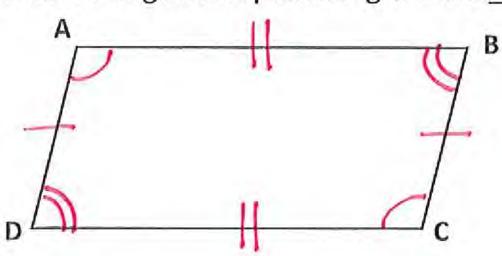
Summary of the Parallelogram Theorems

Opposite sides in a parallelogram are congruent.

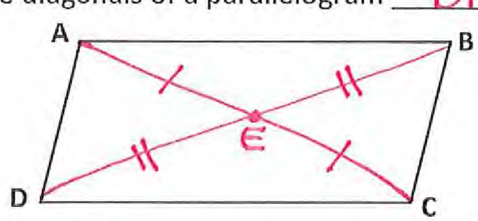
Opposite angles in a parallelogram are congruent.

Consecutive angles in a parallelogram are supplementary.



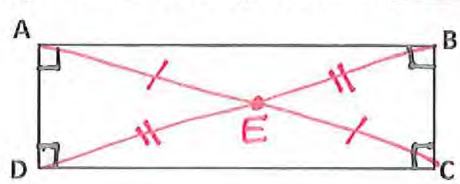
$\angle A + \angle B = 180°$
$\angle D + \angle C = 180°$
$\angle A + \angle D = 180°$
$\angle B + \angle C = 180°$

The diagonals of a parallelogram bisect each other.



$\overline{AE} \cong \overline{EC}$
$\overline{BE} \cong \overline{ED}$

Rectangles are parallelograms with congruent diagonals.



$\overline{AC} \cong \overline{BD}$

$\overline{AE} \cong \overline{EC}$
$\overline{BE} \cong \overline{ED}$