Similarity

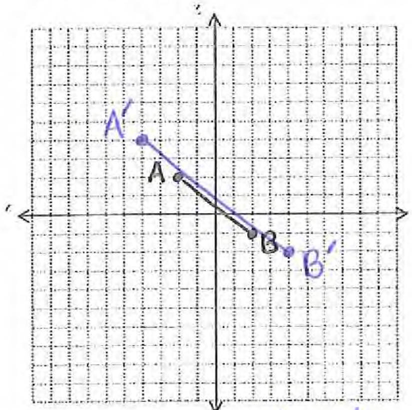
A dilation is a type of transformation that changes the size of a figure. The scale factor measures how much larger or smaller the new figure is.

A dilation maps (x, y) to (kx, ky), where $k > 0$, the center of dilation is $(0, 0)$, and the scale factor is k.

Examples:

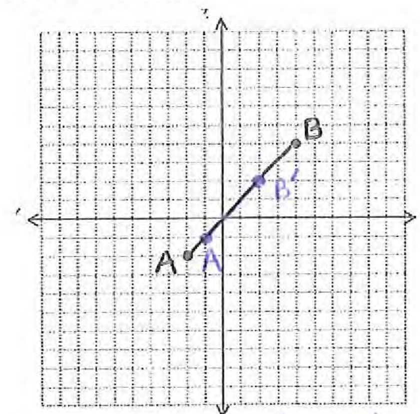
For each example, the center of dilation is at the origin. (a) Dilate the figure by the given scale factor. (b) State the rule that maps each point of the original figure to the corresponding point of the new figure.

1. scale factor: 2



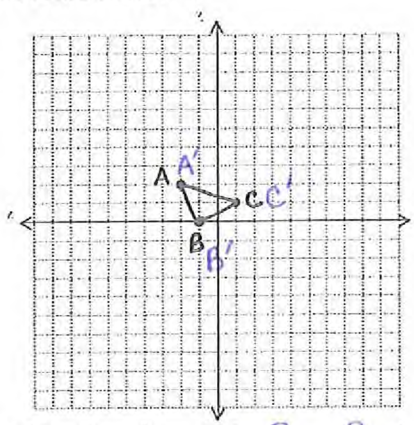
A (-2, 2) → A' (-4, 4)
B (2, -1) → B' (4, -2)
(x, y) → (2x, 2y)

2. scale factor: ½



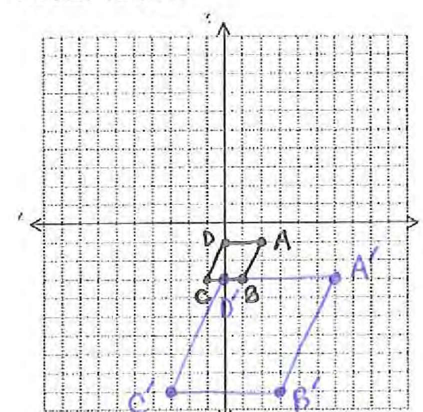
A (-2, -2) → A' (-1, -1)
B (4, 4) → B' (2, 2)
(x, y) → ($\frac{1}{2}$x, $\frac{1}{2}$y)

3. scale factor: 1



A (-2, 2) → A' (-2, 2)
B (-1, 0) → B' (-1, 0)
C (1, 1) → C' (1, 1)
(x, y) → (x, y)

4. scale factor: 3



A (2, -1) → A' (6, -3)
B (1, -3) → B' (3, -9)
C (-1, -3) → C' (-3, -9)
D (0, -1) → D' (0, -3)
(x, y) → (3x, 3y)

When the scale factor is greater than 1, the figure is made larger.

When the scale factor is between 0 and 1, the figure is made smaller.

When the scale factor is 1, the figure does not change.

Examples:

Determine if the figure will be made larger, smaller, or remain the same once it is dilated.

5) A figure is dilated by a scale factor of 1 — remain the same

6) A figure is dilated by a scale factor of 1/5 — smaller

7) A figure is dilated by a factor of 12 — larger

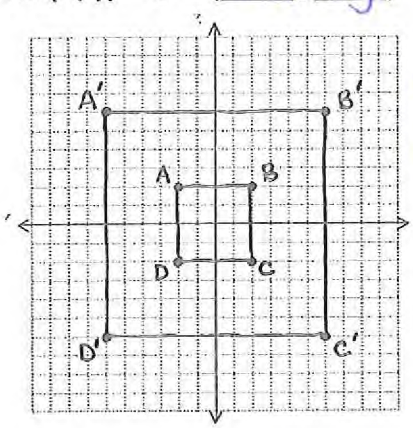
8) A figure is dilated by a factor of 1/3 — smaller

9) A figure is dilated by a factor of 6 — larger

Examples:

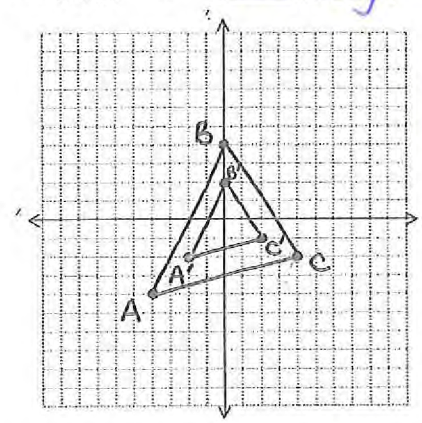
For each example, state the rule that maps each point of the original figure to the corresponding point of the new figure.

10. $(x, y) \rightarrow (3x, 3y)$



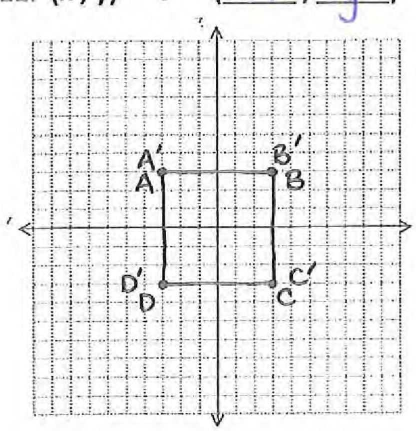
B(2,2)
B'(6,6)

11. $(x, y) \rightarrow (\frac{1}{2}x, \frac{1}{2}y)$

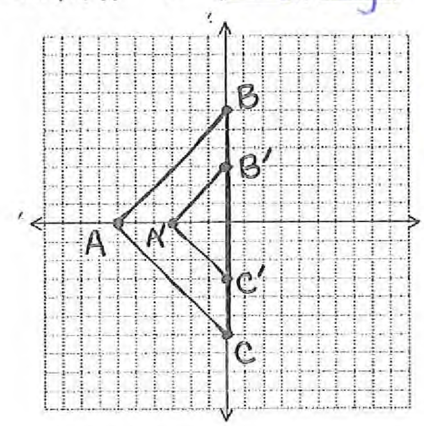


B(0,4)
B'(0,2)

12. $(x, y) \rightarrow (x, y)$



13. $(x, y) \rightarrow (\frac{1}{2}x, \frac{1}{2}y)$



B(0,6)
B'(0,3)

A dilation produces a figure that is the same shape, but a different size than the original figure. The original figure and the dilated figure are similar figures.

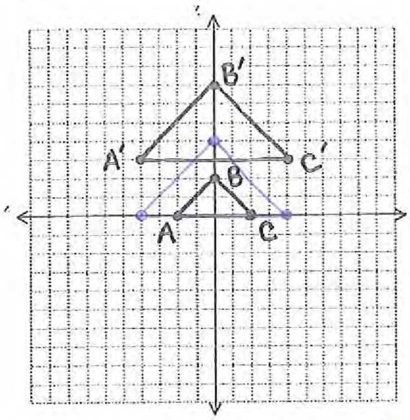
Two figures are similar if there is a sequence of similarity transformations that maps the original figure onto the new figure.

*A similarity transformation is a rigid motion followed by a dilation.

Examples:

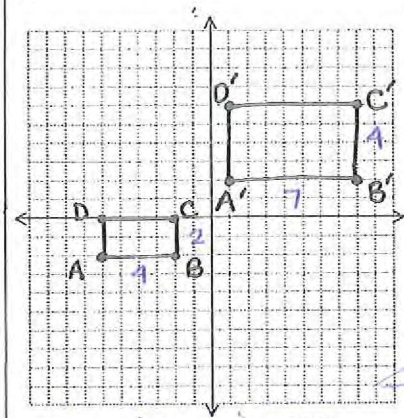
Are the figures similar? If so, state how you know and state the rule that will map each point in the original figure to the corresponding point of the new figure.

14.



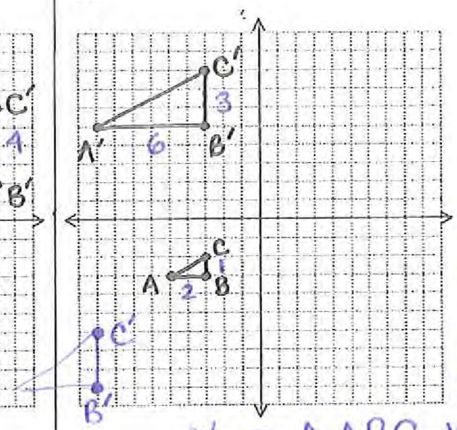
Yes, $\triangle ABC$ was dilated by a factor of 2
$(x,y) \rightarrow (2x,2y)$
and translated up 3 units
$(x,y) \rightarrow (x,y+3)$
to create $\triangle A'B'C'$.

15.



No, there is no sequence of similarity transformations that will create A′B′C′D′ from ABCD.

16.



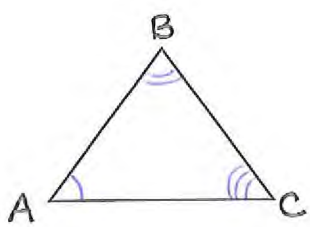
A(-5,-3)
B(-3,-3)
C(-3,-2)

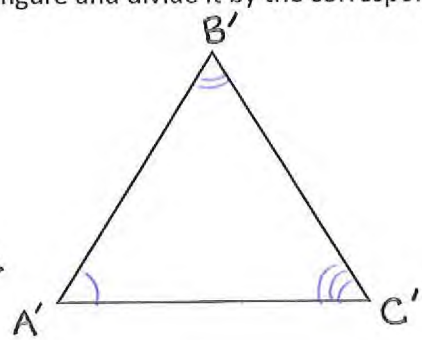
A′(
B′(-9,-9)
C′(-9,-6)

Yes, $\triangle ABC$ was dilated by a factor of 3
$(x,y) \rightarrow (3x,3y)$
and translated up 14 units + right 6 units
$(x,y) \rightarrow (x+6,y+14)$
to create $\triangle A'B'C'$.

If two figures are similar, then their corresponding angles are congruent and their corresponding sides are proportional.

*There is a common scale factor between each pair of corresponding sides. To find scale factor, take a part of the new figure and divide it by the corresponding part of the original figure.





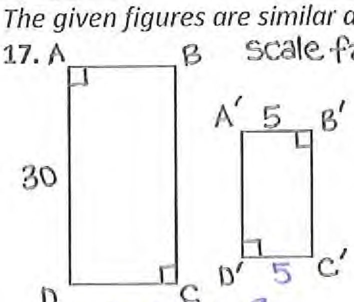
$\angle A \cong \angle A'$
$\angle B \cong \angle B'$
$\angle C \cong \angle C'$

$\frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{A'C'}{AC}$

Examples:

The given figures are similar and you have been given the scale factor. Find the missing measurements.

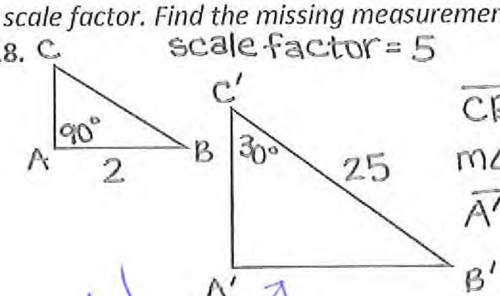
17. scale factor = 1/3



$\times \frac{1}{3}$
$\times 3$

$\overline{CD}$ = 15
$m\angle B$ = 90°
$\overline{A'D'}$ = 10

18. scale factor = 5



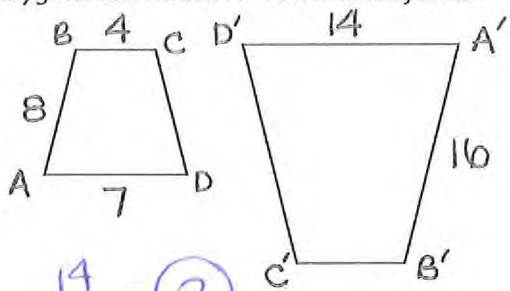
$\times 5$
$\times \frac{1}{5}$

$\overline{CB}$ = 5
$m\angle A'$ = 90°
$\overline{A'B'}$ = 10

Examples:

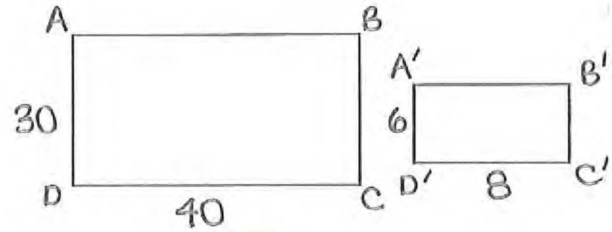
The polygons are similar. Find the scale factor.

19.



$\frac{14}{7} = 2$

20.



$\frac{6}{30} = \frac{1}{5}$

We've talked about the five triangle congruence theorems that can be used to determine if two triangles are congruent, even if we don't know for sure that all 6 pairs of corresponding parts are congruent.

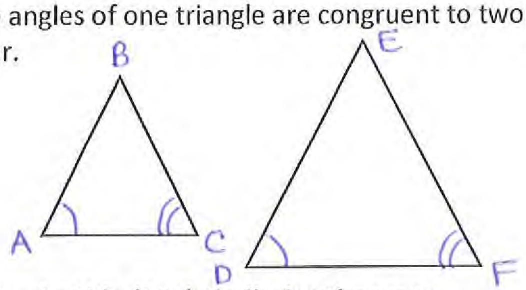
Well...there are triangle similarity theorems as well!

Even if you don't know for sure that all corresponding angles are congruent and that all corresponding sides are proportional in two triangles, you can still determine if the two triangles are similar, using the 3 triangle similarity theorems.

Angle-Angle (AA) Similarity Theorem

If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

If

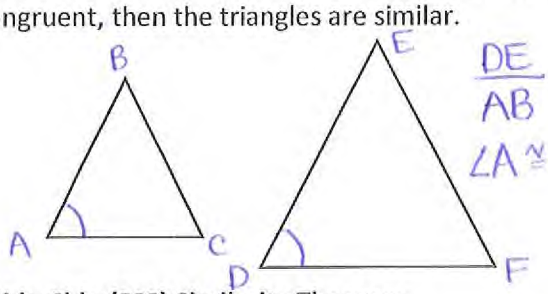


then $\triangle ABC \sim \triangle DEF$.

Side-Angle-Side (SAS) Similarity Theorem

If two sides of one triangle are proportional to two sides of another triangle and their included angles are congruent, then the triangles are similar.

If



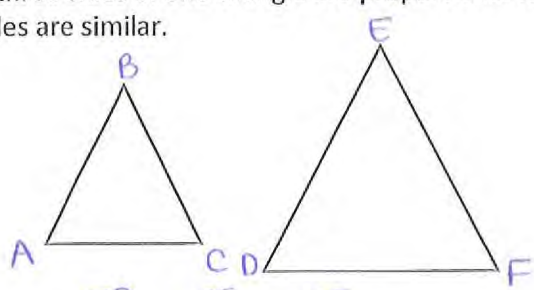
$\frac{DE}{AB} = \frac{DF}{AC}$

$\angle A \cong \angle D$ **then** $\triangle ABC \sim \triangle DEF$.

Side-Side-Side (SSS) Similarity Theorem

If the three sides of one triangle are proportional to the three sides of another triangle, then the two triangles are similar.

If



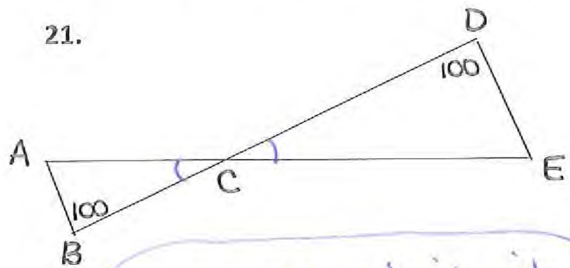
then $\triangle ABC \sim \triangle DEF$.

$\frac{ED}{AB} = \frac{EF}{CB} = \frac{DF}{AC}$

Examples:

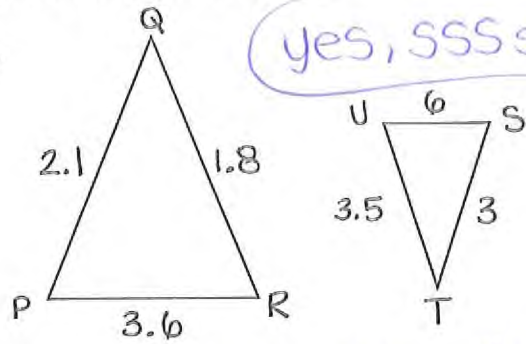
Are the two triangles ~~*congruent*~~ similar*? If so, explain why, showing work when necessary.*

21.



yes, AA similarity

22.

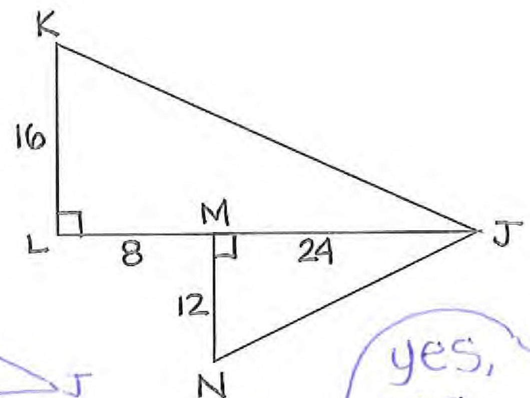


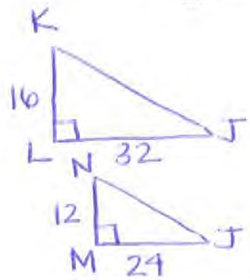
yes, SSS similarity

$\frac{1.8}{3} = \frac{3.6}{6} = \frac{2.1}{3.5}$?

0.6 0.6 0.6 ✓

23.



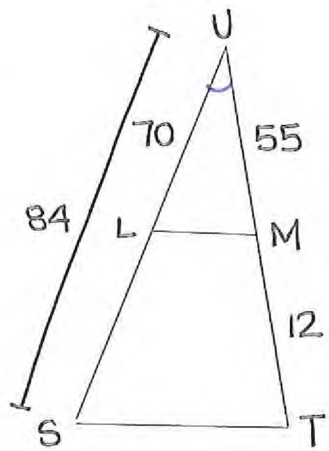


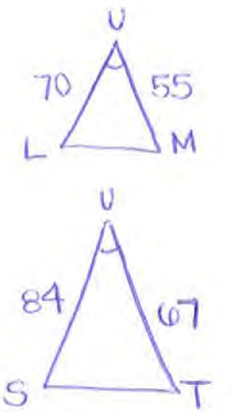
$\frac{32}{24} = \frac{16}{12}$?

$1.\overline{3}$ $1.\overline{3}$ ✓

yes, SAS similarity

24.



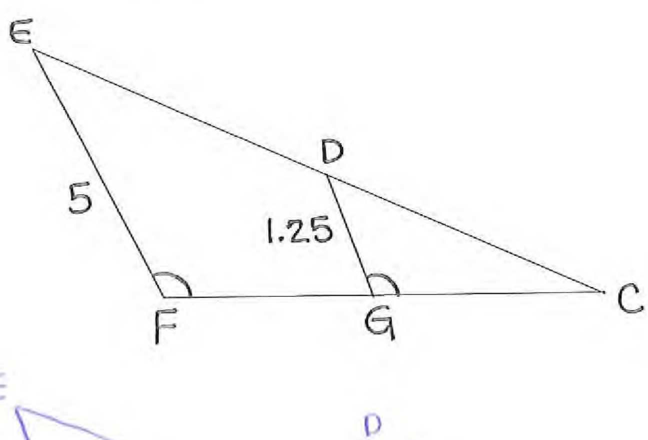


$\frac{84}{70} = \frac{67}{55}$?

1.2 $1.2\overline{18}$ X

not similar

25.



yes, AA similarity