

Solving Quadratics (2)

#29

When can we use the QUADRATIC FORMULA?

when given any quadratic in standard form

QUADRATIC FORMULA:

The solutions of the quadratic equation $ax^2 + bx + c = 0$ are:

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Examples: Find the solutions for each quadratic using the quadratic formula.

1. $x^2 - 5x = 4$

2. $4x^2 + 10x = -10x - 25$

3. $x^2 = 6x + 10$

Examples

$$\text{EX.1) } x^2 - 5x = 4 \rightarrow x^2 - 5x - 4 = 0$$

$-4 \quad -4$ $a=1, b=-5, c=-4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-4)}}{2(1)}$$

$$= \frac{5 \pm \sqrt{25 + 16}}{2} = \frac{5 \pm \sqrt{41}}{2}$$

$$\text{EX.2) } 4x^2 + 10x = -10x - 25 \rightarrow 4x^2 + 20x + 25 = 0$$

$+10x+25 \quad +10x+25$ $a=4, b=20, c=25$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(20) \pm \sqrt{(20)^2 - 4(4)(25)}}{2(4)}$$

$$= \frac{-20 \pm \sqrt{400 - 400}}{8} = \frac{-20 \pm \sqrt{0}}{8} = \frac{-20 \pm 0}{8}$$

$$= \frac{-20}{8} = \frac{-5}{2}$$

$$\text{EX.3) } x^2 = 6x + 10 \rightarrow x^2 - 6x - 10 = 0$$

$-6x-10 \quad -6x-10$ $a=1, b=-6, c=-10$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-10)}}{2(1)}$$

$$= \frac{6 \pm \sqrt{36 + 40}}{2} = \frac{6 \pm \sqrt{76}}{2} = \frac{6 \pm \sqrt{4 \cdot 19}}{2}$$

$$= \frac{6 \pm 2\sqrt{19}}{2} = 3 \pm \sqrt{19}$$

Practice Problems (You may need to work some of the problems on a separate sheet of paper.)

Use the quadratic formula to solve each equation.

1. $x^2 + 4x - 2 = 0$

2. $x^2 + 2x = 4x$

3. $-x^2 + 1 = -5x^2 + 4x$

4. $2x^2 - 5x - 2 = 0$

Solve the equations below using the quadratic formula. Then solve the equation by factoring to check your solutions.

5. $x^2 - 2x - 24 = 0$

6. $6x^2 + 17x + 5 = 0$

REVIEW

Explain how the parent function $y = x^2$ was transformed to create the following quadratics.

7. $y = -(3x + 8)^2 - 12$

8. $y = 5(-x - 3)^2 + 4$

For each quadratic, identify the (a) vertex, (b) axis of symmetry, (c) extrema, (d) y-intercept, (e) zeros, and (f) x-intercepts.

9. $y = 2x^2 - 16x + 30$

10. $y = -3(x + 2)(x - 10)$

Practice Problems

$$1) x^2 + 4x - 2 = 0$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-2)}}{2(1)} = \frac{-4 \pm \sqrt{16 + 8}}{2}$$

$$= \frac{-4 \pm \sqrt{24}}{2} = \frac{-4 \pm \sqrt{4 \cdot 6}}{2} = \frac{-4 \pm 2\sqrt{6}}{2}$$

$$= \boxed{-2 \pm \sqrt{6}}$$

$$2) x^2 + 2x = 4x \rightarrow x^2 - 2x = 0$$

$-4x \quad -4x$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(0)}}{2(1)} = \frac{2 \pm \sqrt{4 - 0}}{2}$$

$$= \frac{2 \pm \sqrt{4}}{2} = \frac{2 \pm 2}{2} = 1 \pm 1 = \boxed{0 \text{ and } 2}$$

$$3) -x^2 + 1 = -5x^2 + 4x \rightarrow 4x^2 - 4x + 1 = 0$$

$+5x^2 - 4x \quad +5x^2 - 4x$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(1)}}{2(4)} = \frac{4 \pm \sqrt{16 - 16}}{8}$$

$$= \frac{4 \pm \sqrt{0}}{8} = \frac{4 \pm 0}{8} = \frac{4}{8} = \boxed{\frac{1}{2}}$$

$$4) 2x^2 - 5x - 2 = 0$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-2)}}{2(2)} = \frac{5 \pm \sqrt{25 + 16}}{4}$$

$$= \frac{5 \pm \sqrt{41}}{4}$$

$$5) x^2 - 2x - 24 = 0$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-24)}}{2(1)} = \frac{2 \pm \sqrt{4 + 96}}{2}$$

$$= \frac{2 \pm \sqrt{100}}{2} = \frac{2 \pm 10}{2} = 1 \pm 5 = (-4 \text{ and } 6)$$

$$\frac{4 \cdot -6}{4} = -24$$

$$\frac{4 + -6}{4} = -2$$

$$y = x^2 + 4x - 6x - 24$$

| | | |
|----|----------------|-----|
| | x | 4 |
| x | x ² | 4x |
| -6 | -6x | -24 |

$$y = (x+4)(x-6)$$

$$x+4=0$$

$$-4 \quad -4$$

$$x = -4$$

$$x-6=0$$

$$+6 \quad +6$$

$$x = 6$$

$$6) 6x^2 + 17x + 5 = 0$$

$$x = \frac{-(17) \pm \sqrt{(17)^2 - 4(6)(5)}}{2(6)} = \frac{-17 \pm \sqrt{289 - 120}}{12}$$

$$= \frac{-17 \pm \sqrt{169}}{12} = \frac{-17 \pm 13}{12} = \frac{-17+13}{12} = \frac{-4}{12} = \frac{-1}{3}$$

$$\frac{-17-13}{12} = \frac{-30}{12} = \frac{-5}{2}$$

$$\underline{2} \cdot \underline{15} = 30$$

$$\underline{2} + \underline{15} = 17$$

$$y = 6x^2 + 2x + 15x + 5$$

| | | |
|----|-----------------|----|
| | 3x | 1 |
| 2x | 6x ² | 2x |
| 5 | 15x | 5 |

$$y = (3x+1)(2x+5)$$

$$3x+1=0$$

$$-1 \quad -1$$

$$\frac{3x}{3} = \frac{-1}{3}$$

$$x = -\frac{1}{3}$$

$$2x+5=0$$

$$-5 \quad -5$$

$$\frac{2x}{2} = \frac{-5}{2}$$

$$x = -\frac{5}{2}$$

$$7) y = -(3x+8)^2 - 12$$

reflect over x-axis

horizontal shrink by $\frac{1}{3}$

left 8

down 12

$$8) y = 5(-x-3)^2 + 4$$

vertical stretch by 5

reflect over y-axis

right 3

up 4

$$9) y = 2x^2 - 16x + 30$$

$$\frac{-6 \cdot -10 = 60}{-6 + -10 = -16}$$

| | | |
|-----|-----------------|-----|
| | x | -3 |
| 2x | 2x ² | -6x |
| -10 | -10x | 30 |

$$y = (x-3)(2x-10)$$

$$x-3=0 \quad 2x-10=0$$

$$+3 \quad +3 \quad +10 \quad +10$$

$$x=3 \quad \frac{2x=10}{2} \quad x=5$$

$$x = \frac{-(-16)}{2(2)} = \frac{16}{4} = 4$$

$$y = 2(4)^2 - 16(4) + 30$$

$$= 32 - 64 + 30$$

$$= -32 + 30$$

$$= -2$$

Vertex: (4, -2)

AOS: x=4

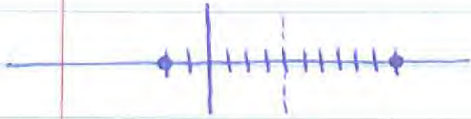
extrema: min at -2

y-int: (0, 30)

zeros: 3, 5

x-intercepts: (3, 0), (5, 0)

$$10) y = -3(x+2)(x-10) \quad -2, 10$$



$$x = 4$$

$$\begin{aligned} y &= -3(4+2)(4-10) \\ &= -3(6)(-6) \\ &= 108 \end{aligned}$$

$$\text{Vertex: } (4, 108)$$

$$\text{AOS: } x = 4$$

extrema: max at 108

$$\text{y-int: } (0, 60)$$

$$\text{zeros: } -2, 10$$

$$\text{x-int: } (-2, 0), (10, 0)$$

$$\begin{aligned} y &= -3(0+2)(0-10) \\ &= -3(2)(-10) \\ &= 60 \end{aligned}$$