Transformations of Quadratics

#3

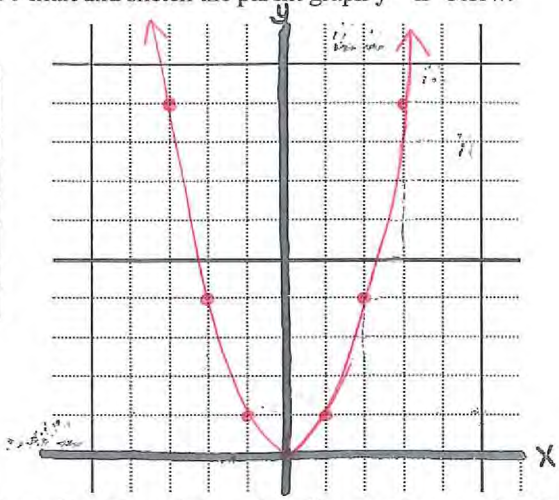
A quadratic function is a function that can be written in the form $y = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$.

- The graph of a quadratic is a parabola.
- The parent function for all quadratics is $y = x^2$.

You will graph various functions and make conjectures based on the patterns you observe from the original function $y=x^2$
Follow the directions below and answer the questions that follow.

- Fill in the t-chart and sketch the parent graph $y = x^2$ below.

x	$y = x^2$
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9



- Now, for each set of problems below, describe what happened to the graph ($y_1 = x^2$) to get the new functions.

Equation	Changes to parent graph.
$y_1 = x^2$ $y_2 = x^2 + 3$ $y_3 = x^2 + 7$	moves up 3 moves up 7

1. Conjecture: The graph of $y = x^2 + a$ will cause the parent graph to move up

Equation	Changes to parent graph.
$y_1 = x^2$ $y_2 = x^2 - 3$ $y_3 = x^2 - 7$	moves down 3 moves down 7

2. Conjecture: The graph of $y = x^2 - a$ will cause the parent graph to move down

Equation	Changes to parent graph.
$y_1 = x^2$ $y_2 = (x+3)^2$ $y_3 = (x+7)^2$	moves left 3 moves left 7

3. Conjecture: The graph of $y = (x + a)^2$ will cause the parent graph to move left

Equation	Changes to parent graph.
$y_1 = x^2$ $y_2 = (x-3)^2$ $y_3 = (x-5)^2$	moves right 3 moves right 5

4. Conjecture: The graph of $y = (x - a)^2$ will cause the parent graph to move right

Equation	Changes to parent graph.
$y_1 = x^2$ $y_2 = -x^2$	flips over x-axis

5. Conjecture: Multiplying the parent graph by a negative causes the parent graph to reflect over x-axis

For the following graphs, please use the descriptions "vertical stretch" (skinny) or "vertical shrink" (fat).

Equation	Changes to parent graph.
$y_1 = x^2$ $y_2 = 3x^2$ $y_3 = 7x^2$	vertical stretch

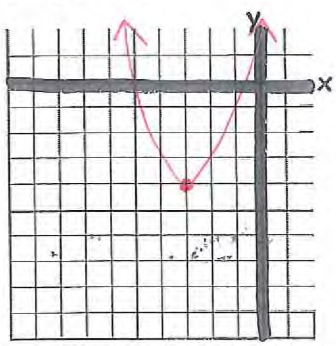
6. Conjecture: Multiplying the parent graph by a number whose absolute value is greater than one causes the parent graph to stretch.

Equation	Changes to parent graph.
$y_1 = x^2$ $y_2 = \frac{1}{2}x^2$ $y_3 = \frac{1}{4}x^2$	vertical shrink

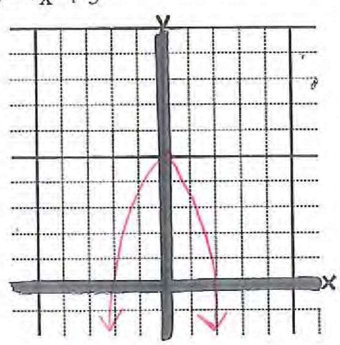
7. Conjecture: Multiplying the parent graph by a number whose absolute value is between zero and one causes the parent graph to shrink.

Based on your conjectures above, sketch the graphs without using your graphing calculator.

8. $y = (x+3)^2 - 4$



9. $y = -x^2 + 5$



Now, go back and graph these on your graphing calculator and see if you were correct. Were you?

Based on your conjectures, write the equations for the following transformations to $y=x^2$.

10. Translated 6 units up $y = x^2 + 6$

11. Translated 2 units right $y = (x-2)^2$

12. Stretched vertically by a factor of 3

$y = 3x^2$

13. Reflected over the x-axis, 2 units left and down 5 units

$y = -(x+2)^2 - 5$

TRANSFORMATIONS OF QUADRATICS

Translations

- slide up $y = x^2 + a$
- slide down $y = x^2 - a$
- slide left $y = (x + a)^2$
- slide right $y = (x - a)^2$

Reflections

- flips across x-axis
  $y = -x^2$
  * parabola opens down
- flips across y-axis
  $y = (-x)^2 = x^2$
  * parabola opens up

Vertical Stretch/Shrink

$y = ax^2$

Stretch
(a is greater than 1)
* pulls points away from x-axis

Ex: $y = 3x^2$
Vertical stretch by a factor of 3

Shrink
(a is between 0 and 1)
* pushes points toward x-axis

Ex: $y = \frac{1}{3}x^2$
Vertical shrink by a factor of $\frac{1}{3}$

Horizontal Stretch/Shrink

$y = (ax)^2$

Stretch
(a is between 0 and 1)
* pulls points away from y-axis

Ex: $y = (\frac{1}{4}x)^2$
Horizontal stretch by a factor of 4

Shrink
(a is greater than 1)
* pushes points toward y-axis

Ex: $y = (4x)^2$
Horizontal shrink by a factor of $\frac{1}{4}$

In class Practice

Explain how the graph of the parent function $y=x^2$ was transformed to create the following functions.

1) $y=5x^2+7$ vertical stretch by 5, move up 7

2) $y=-(3x+9)^2$ reflect over x-axis
horizontal shrink by $\frac{1}{3}$
move left 9

3) $y=(-x)^2-10$ reflect over y-axis
move down 10

4) $y=(\frac{1}{6}x)^2$ horizontal stretch by 6

5) $y=\frac{1}{4}(x-8)^2+2$ vertical shrink by $\frac{1}{4}$
move right 8
move up 2