Unit 5 Practice #6

Name: ______________________________ **#39**

Show your work for ALL problems on a separate sheet of paper.

1. Jason jumped off of a cliff into the ocean in Acapulco while vacationing with some friends. His height as a function of time could be modeled by the function $h(t) = -16t^2 + 16t + 480$, where t is the time in seconds and h is the height in feet. [If necessary, round to the nearest hundredth.]

a) How long did it take for Jason to reach his maximum height?

b) What was the highest point that Jason reached?

c) At what time was Jason's height 200 ft?

d) How long did it take for Jason hit the ocean?

2. If a toy rocket is launched vertically upward from ground level with an initial velocity of 128 feet per second, then its height h after t seconds is given by the equation $h(t) = -16t^2 + 128t$ (if air resistance is neglected). [If necessary, round to the nearest hundredth.]

a) How long will it take for the rocket to return to the ground?

b) After how many seconds will the rocket be 156 feet above the ground?

c) How long will it take the rocket to hit its maximum height?

d) What is the maximum height?

3. Find the rate of change of the function $y = -2(x + 8)^2 - 5$ over the interval $-4 \le x \le -1$.

4. Find the rate of change of the function $y = (x + 2)(x - 6)$ over the interval $2 \le x \le 5$.

5. Find the rate of change of the function $y = -x^2 - 5x + 3$ over the interval $-2 \le x \le 0$.

6. Solve the quadratic $y = -x^2 + 6x$.

7. Solve the quadratic $y = -x^2 + 2x + 3$.

8. Solve the quadratic $y = 4(x + 6)(x - 2)$.

9. Solve the quadratic $8x^2 - 82 = -10$.

10. Solve the quadratic $y = 4(x + 9)^2 - 120$.

11. Identify the vertex, AOS, extrema, y-intercept, x-intercept, zeros, and rate of change over the interval $0 \le x \le 2$ for the table below.

x	-1	0	1	2	3	4	5	6	7
y	0	-5	-8	-9	-8	-5	0	7	16

12. Identify the vertex, AOS, extrema, y-intercept, x-intercept, zeros, and rate of change over the interval $1 \le x \le 3$ for the table below.

x	-5	-4	-3	-2	-1	0	1	2	3
y	5	0	-3	-4	-3	0	5	12	21

① $h(t) = -16t^2 + 16t + 480$

a) (x-coordinate of vertex)

$$x = \frac{-b}{2a} = \frac{-16}{2(-16)} = \frac{-16}{-32} = 0.5 \text{ sec}$$

b) (extrema)

$$y = -16(0.5)^2 + 16(0.5) + 480$$
$$= -16(0.25) + 8 + 480$$
$$= -4 + 8 + 480$$
$$= 484 \text{ ft}$$

c) (Find t when h=200.)

$$200 = -16t^2 + 16t + 480$$
$$0 = -16t^2 + 16t + 280$$
$$\frac{-(16) \pm \sqrt{(16)^2 - 4(-16)(280)}}{2(-16)}$$
$$\frac{-16 \pm \sqrt{256 + 17920}}{-32}$$
$$\frac{-16 \pm \sqrt{18176}}{-32} = -3.71,\ 4.71$$

4.71 sec

d) (Find t when h=0.)

$$0 = -16t^2 + 16t + 480$$
$$\frac{-(16) \pm \sqrt{(16)^2 - 4(-16)(480)}}{2(-16)}$$
$$\frac{-16 \pm \sqrt{256 + 30720}}{-32}$$
$$\frac{-16 \pm \sqrt{30976}}{-32} = \frac{-16 \pm 176}{-32} = -5,\ 6$$

6 sec

② $h(t) = -16t^2 + 128t$

a) (Find t when h=0)

$$0 = -16t^2 + 128t$$
$$\frac{-(128) \pm \sqrt{(128)^2 - 4(-16)(0)}}{2(-16)}$$
$$\frac{-128 \pm \sqrt{16384 + 0}}{-32} = \frac{-128 \pm \sqrt{16384}}{-32}$$
$$\frac{-128 \pm 128}{-32} = 0,\ 8$$

8 sec

b) (Find t when h=156.)

$$156 = -16t^2 + 128t$$
$$0 = -16t^2 + 128t - 156$$
$$\frac{-(128) \pm \sqrt{(128)^2 - 4(-16)(-156)}}{2(-16)}$$
$$\frac{-128 \pm \sqrt{16384 - 9984}}{-32}$$
$$\frac{-128 \pm \sqrt{6400}}{-32} = \frac{-128 \pm 80}{-32} = 1.5,\ 6.5$$

1.5 sec

c) (x-coordinate of vertex)

$$x = \frac{-b}{2a} = \frac{-(128)}{2(-16)} = \frac{-128}{-32} = 4 \text{ sec}$$

d) (extrema)

$$y = -16(4)^2 + 128(4)$$
$$= -16(16) + 512$$
$$= -256 + 512$$
$$= 256 \text{ ft}$$

(3) $y = -2(x+8)^2 - 5, \; -4 \le x \le -1$

$a = -4, \; b = -1$

$f(a) = -2(-4+8)^2 - 5 = -37$

$f(b) = -2(-1+8)^2 - 5 = -103$

$$\frac{f(b)-f(a)}{b-a} = \frac{-103-(-37)}{-1-(-4)} = \frac{-66}{3} = -22$$

(4) $y = (x+2)(x-6), \; 2 \le x \le 5$

$a = 2, \; b = 5$

$f(a) = (2+2)(2-6) = -16$

$f(b) = (5+2)(5-6) = -7$

$$\frac{f(b)-f(a)}{b-a} = \frac{-7-(-16)}{5-2} = \frac{9}{3} = 3$$

(5) $y = -x^2 - 5x + 3, \; -2 \le x \le 0$

$a = -2, \; b = 0$

$f(a) = -(-2)^2 - 5(-2) + 3 = 9$

$f(b) = -(0)^2 - 5(0) + 3 = 3$

$$\frac{f(b)-f(a)}{b-a} = \frac{3-9}{0-(-2)} = \frac{-6}{2} = -3$$

(6) $y = -x^2 + 6x$

$0 = -x^2 + 6x$

$$\frac{-(6) \pm \sqrt{6^2 - 4(-1)(0)}}{2(-1)}$$

$$\frac{-6 \pm \sqrt{36+0}}{-2} = \frac{-6 \pm \sqrt{36}}{-2}$$

$$\frac{-6 \pm 6}{-2} = 0, 6$$

(7) $y = -x^2 + 2x + 3$

$\underline{3} \cdot \underline{-1} = -3$ $\quad -x^2 + 3x - x + 3$

$\underline{3} + \underline{-1} = 2$ $\quad (x+1)(-x+3)$

	$-x$	3
x	$-x^2$	$3x$
1	$-x$	3

$x + 1 = 0$
$\;\;-1 \quad -1$
$x = -1$

$-x + 3 = 0$
$\;\;-3 \quad -3$
$\frac{-x}{-1} = \frac{-3}{-1}$
$x = 3$

OR

$$\frac{-(2) \pm \sqrt{(2)^2 - 4(-1)(3)}}{2(-1)}$$

$$\frac{-2 \pm \sqrt{4+12}}{-2} = \frac{-2 \pm \sqrt{16}}{-2}$$

$$\frac{-2 \pm 4}{-2} = -1, 3$$

(8) $y = 4(x+6)(x-2)$

$x + 6 = 0$
$\;\;-6 \quad -6$
$x = -6$

$x - 2 = 0$
$\;\;+2 \quad +2$
$x = 2$

(9) $8x^2 - 82 = -10$
$\quad +82 \quad +82$

$\frac{8x^2}{8} = \frac{72}{8}$

$\sqrt{x^2} = \sqrt{9}$

$x = \pm 3$

(10) $y = 4(x+9)^2 - 120$

$0 = 4(x+9)^2 - 120$
$+120 \quad\quad +120$

$\frac{120}{4} = \frac{4(x+9)^2}{4}$

$\sqrt{30} = \sqrt{(x+9)^2}$

$\pm\sqrt{30} = x + 9$
$-9 \quad\quad -9$

$-9 \pm \sqrt{30} = x$

(11) vertex: $(2, -9)$

AOS: $x = 2$

ext: min at -9

y-int: $(0, -5)$

x-int: $(-1, 0), (5, 0)$

zeros: $-1, 5$

RoC, $0 \le x \le 2$: -2

$$\frac{f(b)-f(a)}{b-a} = \frac{-9-(-5)}{2-0} = \frac{-4}{2}$$

(12) vertex: $(-2, -4)$

AOS: $x = -2$

ext: min at -4

y-int: $(0, 0)$

x-int: $(-4, 0), (0, 0)$

zeros: $-4, 0$

RoC, $1 \le x \le 3$: 8

$$\frac{f(b)-f(a)}{b-a} = \frac{21-5}{3-1} = \frac{16}{2} = 8$$