

Introduction to Proofs

Properties of Equality

Addition Property	If $a = b$, then $a + c = b + c$.
Subtraction Property	If $a = b$, then $a - c = b - c$.
Multiplication Property	If $a = b$, then $ac = bc$.
Division Property	If $a = b$, then $a \div c = b \div c$.
Reflexive Property	$a = a$
Symmetric Property	If $a = b$, then $b = a$.
Transitive Property	If $a = b$ and $b = c$, then $a = c$.
Substitution Property	If $a = b$, then b can be substituted for a in any expression.
Distributive Property	$a(b + c) = ab + ac$

Example 1: Write the property that each statement represents.

- If $JK = PQ$ and $PQ = ST$, then $JK = ST$. Transitive
- If $m < S = 30$, then $5 + m < S = 35$. Addition Prop
- If $ST = 2$ and $SU = ST + 3$, then $SU = 5$. substitution Prop
- If $m < K = 45$, then $3(m < K) = 135$. Multiplication Prop
- If $m < P = m < Q$, then $m < Q = m < P$. Symmetric Prop

Example 2: Use the property given to complete the statement.

- Symmetric Property: If $m < A = m < B$, then $m < B = m < A$
- Transitive Property: If $BC = CD$ and $CD = EF$, then $BC = EF$
- Substitution Property: If $LK + JM = 12$ and $LK = 2$, then $2 + JM = 12$
- Subtraction Property: If $PQ + ST = RS + ST$, then $PQ = RS$
- Division Property: If $3(m < A) = 90$, then $m < A =$ 30

Each statement in a proof must follow logically from what has come before and must have a reason to support it. The reason may be a piece of given information, a definition, a previously proven theorem, or a mathematical property.

Example 3:

Given: $4x - 8 = -12$

Prove: $x = -1$

Statement	Reason
$4x - 8 = -12$	Given
<u>$4x = -4$</u>	Addition Prop of Equality
<u>$x = -1$</u>	<u>Division Prop of Equality</u>

Example 4:

Given: $2(3x + 1) = 5x + 14$

Prove: $x = 12$

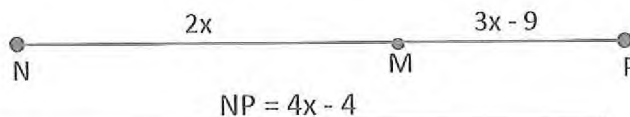
Statement	Reason
<u>$2(3x + 1) = 5x + 14$</u>	Given
<u>$6x + 2 = 5x + 14$</u>	<u>Distributive Prop</u>
<u>$x + 2 = 14$</u>	<u>Subtraction Prop</u>
<u>$x = 12$</u>	<u>Substitution Prop of Equality</u> <u>subtraction</u>

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Example 5:

Given: M is in between N and P.

Prove: $x = 5$

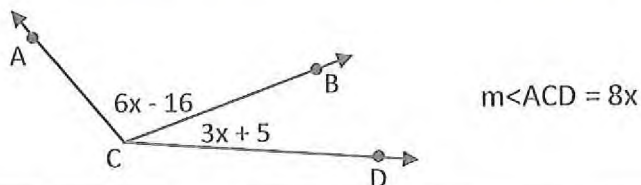


Statement	Reason
M is in between N and P.	Given
$NM + MP = NP$	Segment Addition Postulate
$2x + 3x - 9 = 4x - 4$	Substitution Property of Equality
$5x - 9 = 4x - 4$	Simplify
$5x = 4x + 5$	Addition Prop of Equality
$x = 5$	Subtraction Prop of Equality

Example 6:

Given: CB lies in the interior of $\angle ACD$.

Prove: $x = 11$

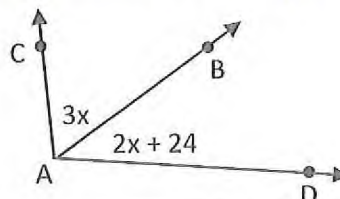


Statement	Reason
\overrightarrow{CB} lies in the interior of $\angle ACD$	Given
$m\angle ACB + m\angle BCD = m\angle ACD$	Angle Addition Postulate
$6x - 16 + 3x + 5 = 8x$	Substitution Property of Equality
$9x - 11 = 8x$	Simplify
$5x = 4x + 5$	Subtraction Prop of Equality
$x - 11 = 0$	Addition Prop of Equality
$x = 11$	

Example 7:

Given: AB is the angle bisector of $\angle CAD$.

Prove: $x = 24$

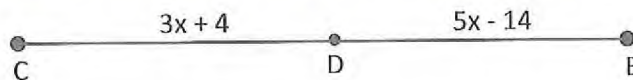


Statement	Reason
\overrightarrow{AB} is the angle bisector of $\angle CAD$	Given
$m\angle CAB = m\angle BAD$	Def of Angle Bisector
$3x = 2x + 24$	Substitution Prop of Equality
$x = 24$	Subtraction Prop of Equality

Example 8:

Given: D is the midpoint of \overline{CB} .

Prove: $x = 9$



Statement	Reason
D is the midpoint of \overline{CB}	Given
$CD = DB$	Def of Midpoint
$3x + 4 = 5x - 14$	Substitution Prop of Equality
$4 = 2x - 14$	Subtraction Prop of Equality
$18 = 2x$	Addition Prop of Equality
$9 = x$	Division Prop of Equality
$x = 9$	Symmetric Prop of Equality