Geometric Constructions (MCC9-12.G.CO.12-13, MCC9-12.G.CO.10)

Introduction

In this task, students will investigate and perform geometric constructions using Euclidean tools. The focus of this task is to learn how to copy line segments, copy an angle, bisect a segment, bisect an angle, construct perpendicular lines (including the perpendicular bisector of a line segment), construct a line parallel to a given line through a point not on the line, and construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

The study of Geometry was born in Ancient Greece, where mathematics was thought to be embedded in everything from music to art to the governing of the universe. Plato, an ancient philosopher and teacher, had the statement, "Let no man ignorant of geometry enter here," placed at the entrance of his school. This illustrates the importance of the study of shapes and logic during that era. Everyone who learned geometry was challenged to construct geometric objects using two simple tools, known as Euclidean tools: a straight edge without any markings and a compass.

The straight edge could be used to construct lines, the compass to construct circles. As geometry grew in popularity, math students and mathematicians would challenge each other to create constructions using only these two tools. Some constructions were fairly easy (Can you construct a square?), some more challenging (Can you construct a regular pentagon?), and some impossible even for the greatest geometers (Can you trisect an angle?).

What constructions can you create??

Challenge 1: Copy a line segment

Step 1. Draw a ray that is longer than the given line segment.
Step 2. Use your compass to measure the length of the given line segment.
Step 3. Without changing the width of the compass, place the point of the compass at the endpoint of the ray that you constructed and draw an arc on the ray.
Step 4. At the place where the arc intersects with the ray, draw a point. Use the original endpoint of the ray and the point that you have just constructed to draw the segment.

Examples: Copy each line segment using only a straight edge and a compass.

a)

b)

Reflection: Answer the following question(s) using complete sentences.

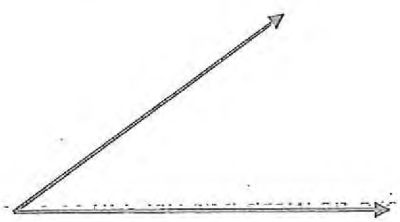
1. Explain how you would copy a segment that is two times the length of the given segment.

Challenge 2: Copy an angle

Step 1. Draw a ray.
Step 2. Place the point of the compass on the vertex of the original angle and draw an arc. (The width of the compass does not matter.)
Step 3. Without changing the width of the compass, place the point of the compass on the endpoint of the ray and draw an arc.
Step 4. Use the compass to measure the distance between the points where the original angle intersects with the arc that you constructed.
Step 5. Without changing the width of the compass, place the point of the compass at the place where the ray intersects with the arc and draw an arc.
Step 6. Using a straight edge, draw a ray that begins at the endpoint of the original ray and goes through the place where the two arcs intersect.

Examples: Copy each angle using only a straight edge and a compass.

a)



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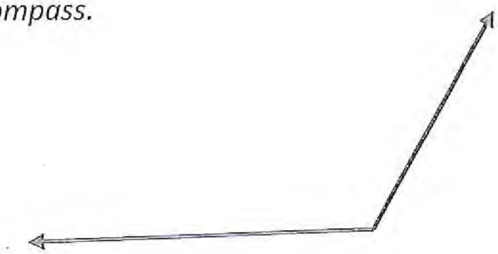


Reflection: Answer the following question(s) using complete sentences.

1. When copying an angle, does the length of the rays of the new angle have to be the same as the length of the rays of the original angle? Explain.

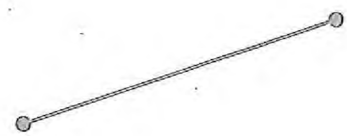
Challenge 3: Bisect a line segment

Step 1. Place the point of the compass at one endpoint of the segment. Adjust the width of the compass so that it is more than half of the segment. Draw a semicircle.
Step 2. Without changing the width of the compass, place the point of the compass on the other endpoint of the segment. Draw a semicircle.
Step 3. Using a straight edge, draw a line that goes through the two points where the semicircles intersect.
**This line is perpendicular to the segment and it splits the segment into two congruent, smaller segments.

Examples: Bisect each line segment using only a straight edge and a compass.

a)

b)



Reflection: Answer the following question(s) using complete sentences.

1. What does it mean to bisect something?

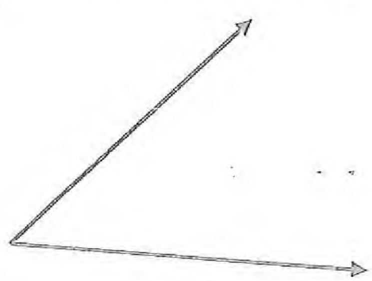
2. Is it possible to bisect a line? Explain.

Challenge 4: Bisect an angle

Step 1. Place the point of the compass on the vertex of the angle and draw an arc.
Step 2. Place the point of the compass on one of the points where the arc intersects with one of the angle's rays. Draw an arc.
Step 3. Without changing the width of the compass, place the point of the compass on the other point where the arc intersects with the other angle ray. Draw an arc.
Step 4. Use a straight edge to draw a ray beginning at the vertex of the angle through the point where the two arcs intersect.

Examples: Bisect each angle using only a straight edge and a compass.

a)



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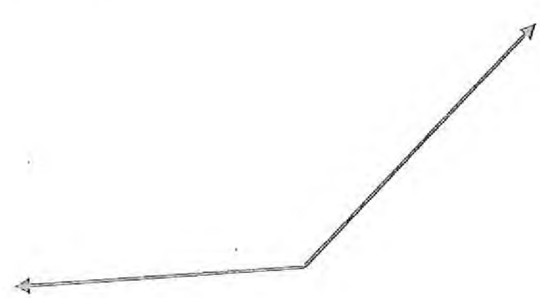
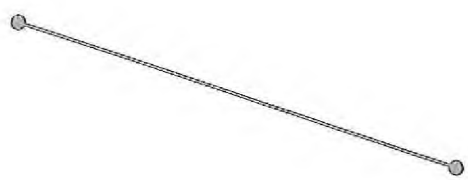


Reflection: Answer the following question(s) using complete sentences.
1. Define angle bisector in your own words.

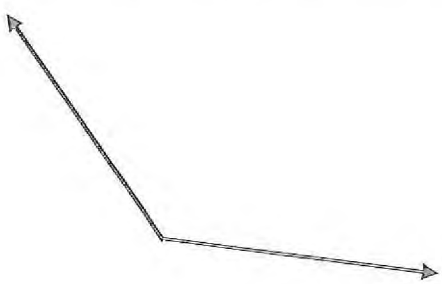
2. How is the bisector of an angle similar to the bisector of a line segment?

Practice: Challenge 1 – 4

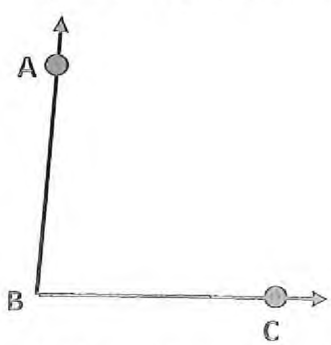
1) Copy the given segment to create segment CD. Then bisect segment CD.



2) Copy the given angle to create angle CED. Then bisect angle CED.



3) Bisect angle ABC to create angle ABD and angle DBC. Then copy angle DBC.



4) Bisect segment AB to create segment AC and segment CB. Then copy segment AC.

Challenge 5: Construct Parallel Lines

Step 1. Draw a transverse line through R and across the line PQ at an angle, forming the point J where it intersects the line PQ. (The exact angle is not important.)

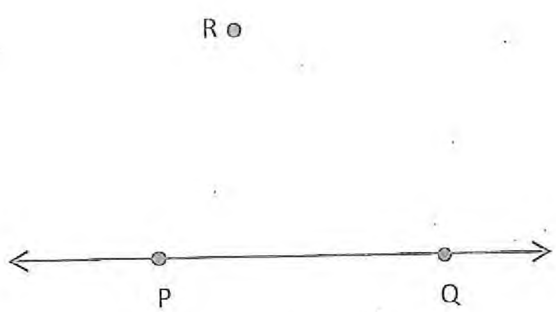
Step 2. With the compass width set to about half the distance between R and J, place the point on J, and draw an arc across both lines.

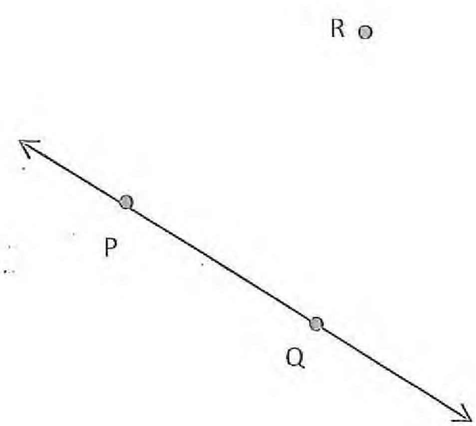
Step 3. Without adjusting the compass width, move the compass to R and draw a similar arc to the one in step 2.

Step 4. Set compass width to the distance where the lower arc crosses the two lines.

Step 5. Move the compass to where the upper arc crosses the transverse line and draw an arc across the upper arc, forming point S.

Step 6. Draw a straight line through points R and S.





Reflection:

1. Constructing parallel lines is similar to what other construction?

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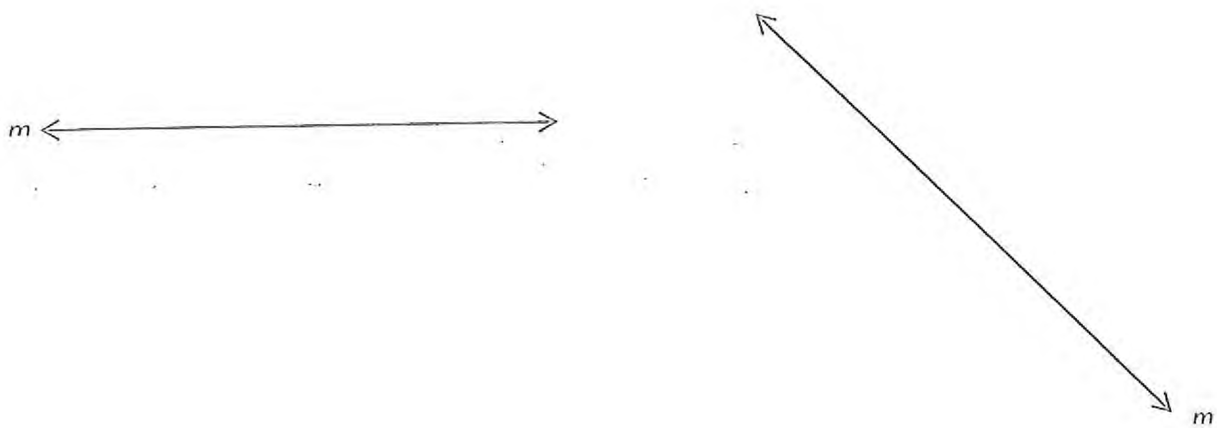
Challenge 6: Construct Perpendicular Lines

Step 1. Draw two points on line m. Label points A and B.
Step 2. Place point on A.
Step 3. Set compass more than half-way of AB. Make a semi-circle arc so that AB is intersected.
Step 4. Without changing the width of your compass, place point on B.
Step 5. Make a semi-circle arc so that the previously drawn arc is intersected at two points.
Step 6. Label the intersection points of the arc points C and D.
Step 7. Use your straightedge to connect points C and D.
Step 8. Label the intersection point of these two lines point K.



Reflection: What is the measure of angle <CKA? ____________________

What is the measure of angle <DKA? ____________________

Will these angles always be equal? Explain. ________________________________

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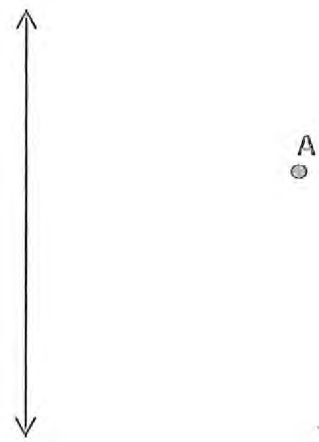
Practice: Challenge 5 – 6

1) Construct a line that is parallel to the given line that goes through point A.

a)



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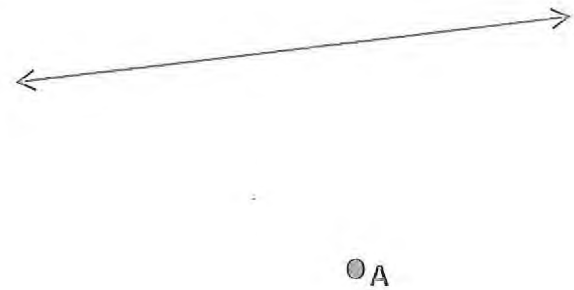


2. Construct a line that is perpendicular to the given line.

a)

b)

Challenge 7: Construct a Regular Hexagon Inscribed in a Circle

Step 1. Draw a circle with center A.

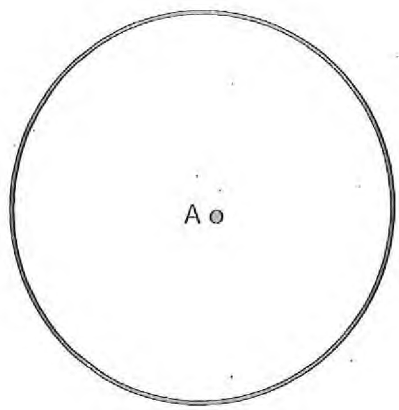
Step 2. Plot point B on the circle.

Step 3. Set compass to width of radius. Place compass on B and draw an arc that intersects the circle.

Step 4. Without changing the width of the compass, place point on intersection point and draw another arc that intersects the circle.

Step 5. Repeat process until you have gone all the way around the circle (three times).

Step 6. Connect every intersection point to form a regular hexagon (You should have 6 points).



Reflection: Is AB the only radius possible for circle A? Explain.

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Challenge 8: Construct an Equilateral Triangle Inscribed in a Circle

Step 1. Draw a circle with center K.

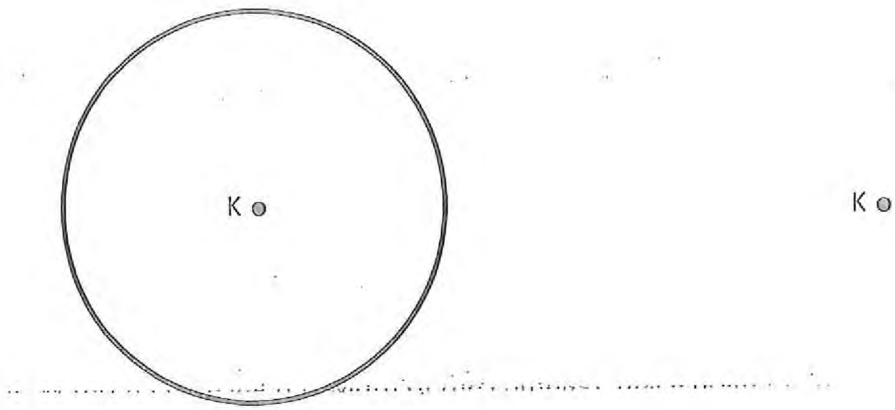
Step 2. Plot point B on the circle.

Step 3. Set compass to width of radius. Place compass on B and draw an arc that intersects the circle.

Step 4. Without changing the width of the compass, place point on intersection point and draw another arc that intersects the circle.

Step 5. Repeat process until you have gone all the way around the circle (three times).

Step 6. Use a straightedge to connect every other point (3 points).



Reflection: Label your three vertices of your triangle D, E, F. What does DE represent in a circle?

Challenge 9: Construct a Square Inscribed in a Circle

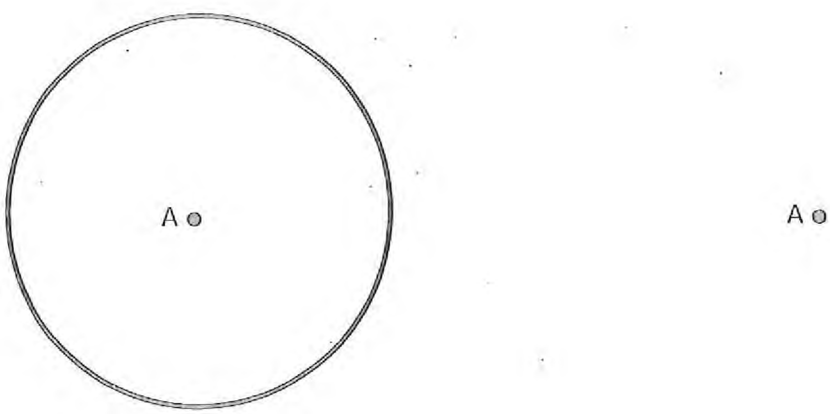
Step 1. Draw a circle with center A.

Step 2. Plot a point anywhere on the circle.

Step 3. Draw diameter through this point. Label endpoints B and C.

Step 4. Construct segment bisector of the diameter. Label the points in which the perpendicular line intersects the circle points D and E.

Step 5. Connect all four points on circle to form square.



Reflection: Is BC the only diameter possible for circle A? Explain. ____________________

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Based on your construction, what is true of the diagonals of a square? ____________

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Practice: Challenge 7-9

1. Construct an equilateral triangle inscribed in a circle.

a) (Draw your own circle.)

b) (Draw your own circle.)

2. Construct a regular hexagon inscribed in a circle.

a) (Draw your own circle.)

b) (Draw your own circle.)

3. Construct a square inscribed in a circle.

a) (Draw your own circle.)

b) (Draw your own circle.)

Challenge 10: Discovering the Centroid of a Triangle

The median of a triangle is a line segment joining a vertex to the midpoint of the opposite side.

Based on that definition, how many medians does a triangle possess?

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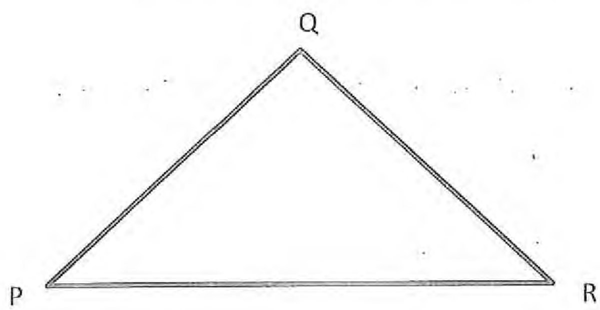
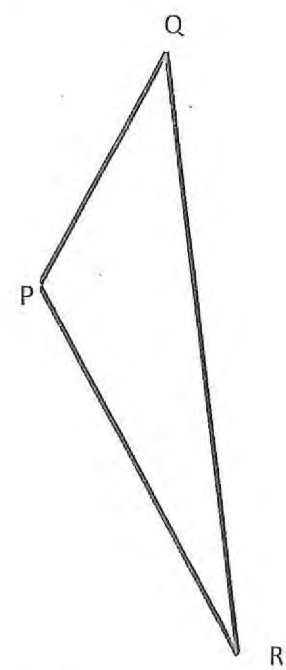
Step 1. Construct the bisector of the line segment PQ. Label the midpoint of the line S.
Step 2. Draw the median from the midpoint S to the opposite vertex R
Step 3. In the same manner, construct T, the midpoint of the line segment QR.
Step 4. Draw the median from the midpoint T to the opposite vertex P
Step 5. In the same manner, construct U, the midpoint of the line segment PR.
Step 6. Label the point where all medians intersect, C. This is called the **centroid.**





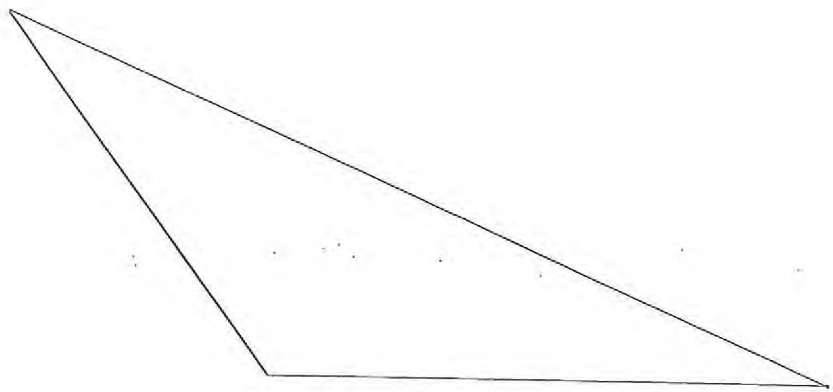
Reflection: Where will the centroid of a circle always be located? _______________

Define centroid of a triangle. _______________

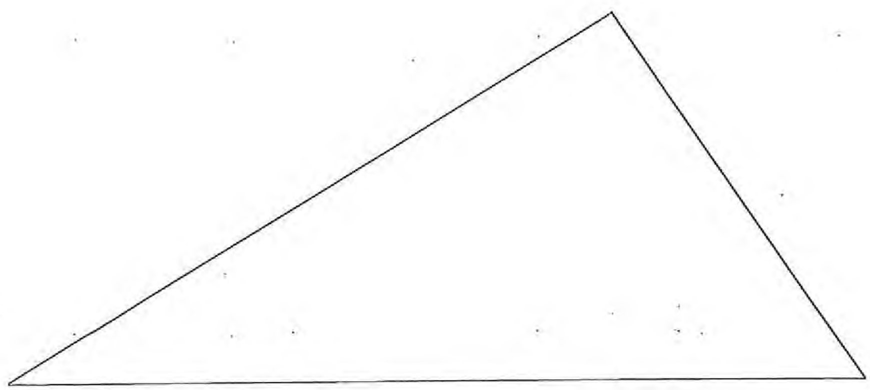
Practice: Challenge 10

Construct the centroid of each triangle.

1)



2)



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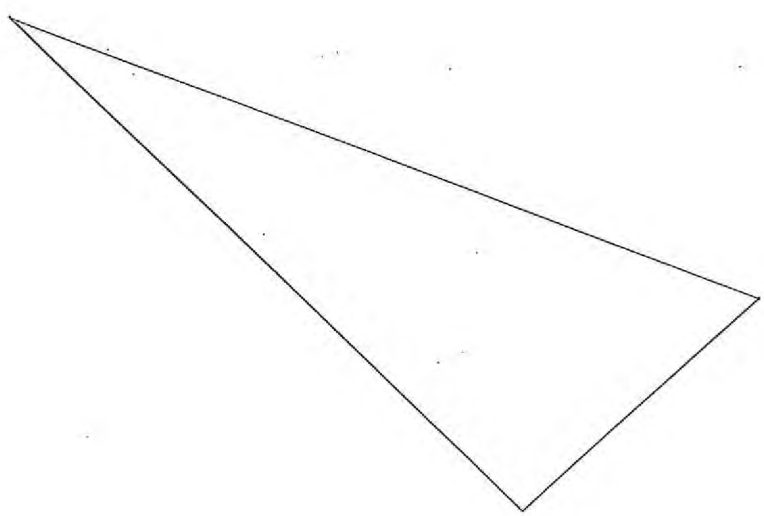
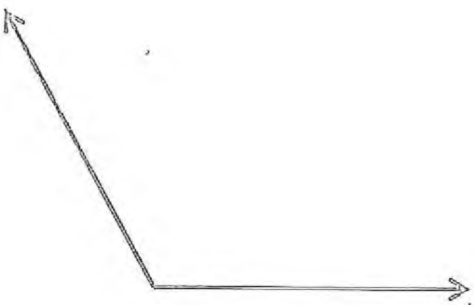
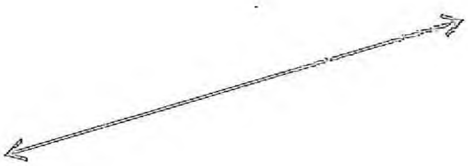


Construction Review
(All Challenges 1-10)

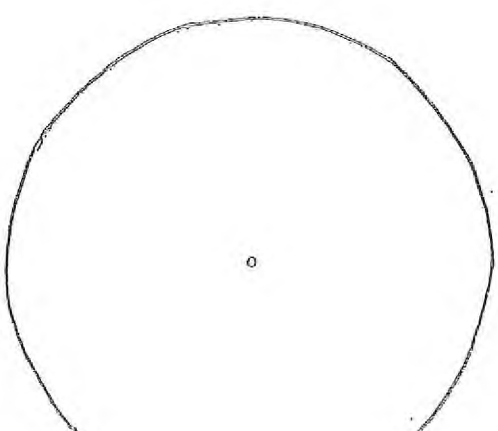
1. Bisect the angle below.



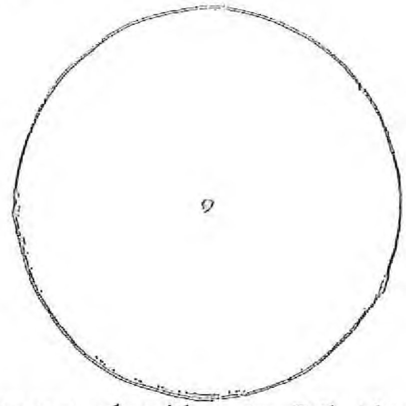
2. Construct a line that is perpendicular to the given line.



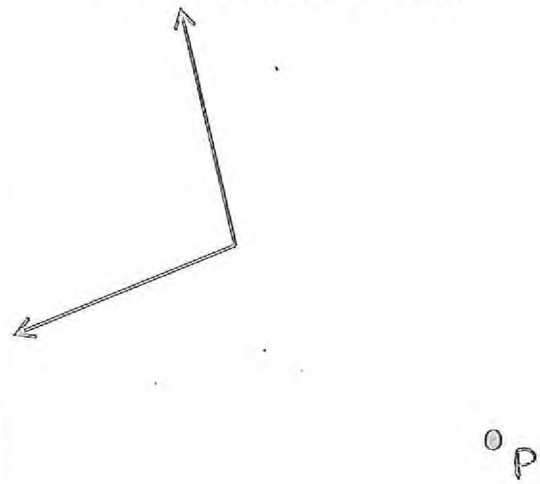
3. Construct a square inscribed in a circle.



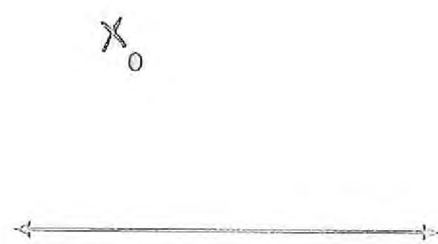
4. Construct a regular hexagon inscribed in a circle.



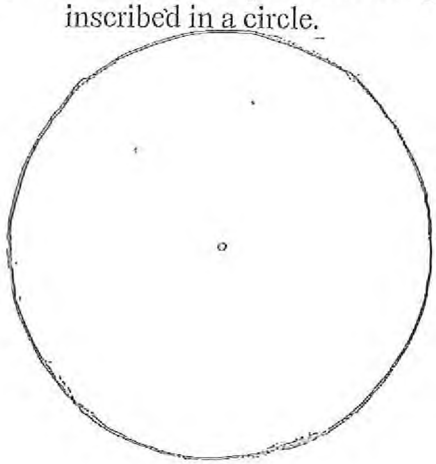
5. Copy an angle with vertex P that has the same measure as the given angle.



6. Construct a line parallel to the given line that goes through point X.

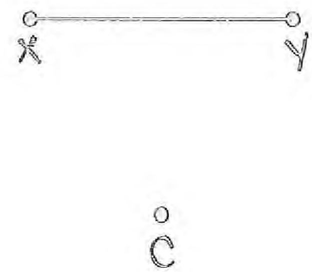


7. Construct an equilateral triangle inscribed in a circle.

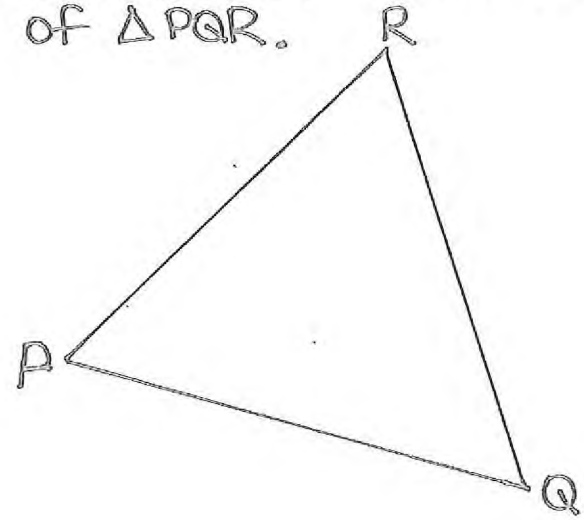


8. Bisect segment LM.

9. Copy a segment CD that is the same length as segment XY.



10. Construct the centroid of ΔPQR.



11. Construct the centroid of ΔLMN.